The Coase Proposition, Information Constraints, and Long-Run Equilibrium: Comment

By HENRY B. HANSMANN*

In a recent paper in this *Review*, William Schulze and Ralph d'Arge employ a partial equilibrium model of two competitive industries with an externality to analyze the well-known "Coase proposition." In particular, they compare both the short-run and long-run efficiency implications of (a) the unadjusted externality case, (b) a Pigovian tax on the output of the firms generating the external cost (the "emitting" firms), (c) a rule making the emitting firms liable to the "receptor" firms, and (d) a situation in which the emitting firms incur no liability, but firms in the two industries are free to bargain costlessly concerning the externality. The analysis in general is illuminating. However, their conclusions regarding the impact of a liability rule, both in the short run and in the long run, appear to be in error. This is of particular significance because, as the authors point out, it is precisely the impact of a liability rule that has been at the center of the controversy over the Coase proposition.

I. The Short Run

According to Schulze and d'Arge, in the short run a liability rule will lead to the optimum level of output in both industries. Under the assumptions they make, however, the liability rule will in fact lead to higher production in the emitting industry than is socially optimal. The error lies in the derivation of their equation (8), which gives the condition for profit maximization in the emitting industry. To see the mistake, it is helpful to rewrite the second equation in their equations (6), which shows the profit for a representative firm in the emitting industry, and from which (8) is derived, as

$$(1) \quad \pi_2 = P_2 y_2 - C_2(y_2) - n_1 D_1(Q_2)/n_2$$

where y_2 is the output of a representative firm inthe emitting industry, Q_2 is the total output of the n_2 firms in the emitting industry, C_2 is the direct cost of producing y_2, and $D_1(Q_2)$ is the cost to each of the n_1 firms in the receptor industry of the externality associated with production level Q_2 in the emitting industry. (This formulation follows the authors in assuming that the amount paid to the receptor industry is divided up equally among the emitting firms.)[1] The first-order condition for profit maximization is then

$$(2) \quad \partial \pi_2 / \partial y_2 = P_2 - C_2' - n_1 D_1' \left[\frac{1}{n_2} \frac{dQ_2}{dy_2} \right] = 0$$

This is the same as their equation (8), except that they assume that $dQ_2/dy_2 = n_2$, and thus that the term in brackets equals unity. That is, their formulation assumes that each firm expects that if it increases output by one unit, so will all other firms in the industry, and total industry output will therefore increase by n_2 units. Since they explicitly assume that the emitting industry is competitive, however, a firm in that industry would behave as if $dQ_2/dy_2 = 1$, or, in other words, as if its decisions had no effect on the behavior of other firms. Thus, we can rewrite (2) as

[1]Alternatively, we could assume that liability is divided up among emitting firms according to their output, so that equation (1) instead appears as

$$\pi_2 = P_2 y_2 - C_2(y_2) - n_1 D_1(Q_2) y_2 / Q_2$$

However, so long as n_2 is large and, as the authors assume, $D'' > 0$ (so that marginal damages exceed average damages), short-run output in the emitting industry under a liability rule will still exceed the social optimum.

*Assistant professor of law, University of Pennsylvania. I wish to thank William Brainard for valuable comments.

$$(2') \quad \partial \pi_2/\partial y_2 = P_2 - C_2' - n_1 D_1'/n_2 = 0$$

This condition will be met for individual firms only when their output—and that for the industry as a whole—exceeds the level corresponding to a social optimum. Indeed, if n_2 is large the short-run equilibrium for the emitting industry will be quite close to that in the unadjusted externality case, in which firms in the emitting industry ignore the impact of the externality altogether. In large part, then, the effect of Schulze and d'Arge's liability rule is not to force internalization of the external costs engendered by an emitting firm, but rather simply to shift the burden of those costs away from the firms in the receptor industry and onto the other firms in the emitting industry.

II. The Long Run

In the long run, the authors assert the liability rule results in an overallocation of resources to both industries. Their conclusion is correct so far as the receptor industry is concerned. In the emitting industry, however, the effect of a liability rule might well be underproduction rather than overproduction.

Schulze and d'Arge's argument is based upon their Figure 2. They correctly note that in long-run equilibrium a representative firm in the emitting industry will operate along an average cost curve given by $AC_2 = C_2/y_2 + n_1 D_1/n_2 y_2$. They argue that this average cost curve will always lie below the curve corresponding to the optimal Pigovian tax because, under their assumptions, average damages (upon which liability payments are based) are always below marginal damages (upon which the optimal tax is based). Therefore, they reason, under a liability rule price will be set too low in the emitting industry, and total output will be too high. Yet, as noted above, they are incorrect in stating that the *marginal* cost curve perceived by an individual firm is given by $C_2' + n_1 D_1'$ rather than by $C_2' + n_1 D_1'/n_2$. Consequently, with a liability rule firms should be expected to produce at an output level exceeding that which corresponds to the lowest point on the long-run average cost curve. Thus simply observing that the minimum on one average cost curve lies below the minimum on the other does not suffice to establish their conclusion.

But more remarkably, while Schulze and d'Arge correctly note that a liability rule will induce entry of firms into the receptor industry beyond the social optimum, they ignore the fact that this increase in n_1 will tend to *raise* both marginal and average cost in the emitting industry by increasing the amount of damages that must be paid. Similarly, they appear to ignore the effect of changes in the number of firms in the emitting industry, n_2, upon the position of the marginal and average cost curves both directly and via the terms D_1 and D_1', both of which depend on n_2.

When all of these factors are accounted for, it is clear that the long-run equilibrium price in the emitting industry could as well be above as below the social optimum, and thus that there could as well be too little as too much production in that industry. In fact, one would expect underproduction to be the typical result. Only if marginal damages exceed average damages by a substantial amount, and demand in the receptor industry is quite price inelastic (so that the reduction in cost and price resulting from receipt of compensation would cause little expansion in that industry, and thus little increase in the liability of the emitting firms), would one expect to find overproduction in the emitting industry under a liability rule.

These points can be illustrated with a simple example. Assume that the cost functions for the two industries are given by

$$C_i = (y_i - 100)^3 + 100^3 \quad i = 1, 2$$
$$D_1 = (n_2 y_2)^2/75$$

and that the demand curves for the products of the two industries are given by

$$P_i = 30{,}000 - n_i y_i/2 \quad i = 1, 2$$

These functions give the conventional U-shaped average cost curves for both industries, and satisfy as well all of the other conditions set out by Schulze and d'Arge.

The conditions for a social optimum are those given by the authors as (their equations (2)–(5)):

$$(3)\qquad P_1 = C_1'$$
$$(4)\qquad P_2 = C_2' + n_1 D_1'$$
$$(5)\qquad P_1 y_1 = C_1 + D_1$$
$$(6)\qquad P_2 y_2 = C_2 + n_1 y_2 D_1'$$

Using the specific cost and demand functions just given, these four equations can be solved (by means of substitution and iterative estimation) for the four unknowns, yielding the (rounded) values[2] $n_1 = 34.6$, $n_2 = 105$, $y_1 = 194$, and $y_2 = 150$. The optimal total output of the emitting industry is therefore $n_2 y_2 = 15{,}750$.

With a liability rule, the long-run equilibrium will be characterized by the four equations

$$(3')\qquad P_1 = C_1'$$
$$(2')\qquad P_2 = C_2' + n_1 D_1'/n_2$$
$$(5')\qquad P_1 y_1 = C_1$$
$$(6')\qquad P_2 y_2 = C_2 + n_1 D_1/n_2$$

which are the same as those given by the authors except for (2′), which is discussed above. Substituting the specific functional forms assumed here, these equations can again be solved for the four unknowns giving $n_1 = 300$, $n_2 = 23.7$, $y_1 = 150$, and $y_2 = 193$. Thus, with a liability rule, the total output of the emitting industry is $n_2 y_2 = 4{,}581$. Rather than being greater than the optimum output for the emitting industry, as Schulze and d'Arge predict, this is in fact less than one-third of the optimum output as calculated above.

[2]The figures given here and in the liability rule case below suggest that firms in each industry are divisible. Only minor adjustments in the figures are necessary if an integral number of firms is required.

REFERENCES

W. Schulze and R. C. d'Arge, "The Coase Proposition, Information Constraints, and Long-Run Equilibrium," *Amer. Econ. Rev.*, Sept. 1974, *64*, 763–72.