

# SYMBOLIC LOGIC: A RAZOR-EDGED TOOL FOR DRAFTING AND INTERPRETING LEGAL DOCUMENTS

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A LARGE amount of the litigation based on written instruments—whether statute, contract, will, conveyance or regulation—can be traced to the draftsman's failure to convey his meaning clearly. Frequently, of course, certain items may purposely be left ambiguous, but often the question in issue is due to an inadvertent ambiguity that could have been avoided had the draftsman clearly expressed what he intended to say. In this Article it is suggested that a new approach to drafting, using certain elementary notions of symbolic logic, can go a long way towards eliminating such inadvertent ambiguity. This new approach makes available to draftsmen a technique that achieves some of the clarity, precision and efficiency of analysis that symbolic logic provides. In addition, it can be a valuable aid in moving towards a more comprehensive and systematic method of interpretation,<sup>1</sup> as well as drafting.

This approach is a compromise between expression in ordinary prose and expression in the mathematical notation of symbolic logic—enough like ordinary prose to be understood easily by any careful reader, enough like symbolic logic to achieve some of its important advantages. It represents an effort to adapt some of the techniques of symbolic logic to make more systematic what is now best described as the “art” of drafting.

The first section will explain six elementary logical connectives: implication, conjunction, coimplication, exclusive disjunction, inclusive disjunction and negation. In order to simplify this exposition, trivial examples will be used for purposes of illustration. In the second section the proposed system will be applied to actual legal problems of drafting, interpretation, simplification and comparison.

## SIX ELEMENTARY LOGICAL CONNECTIVES<sup>2</sup>

### 1.0 *Implication*

The development of a more systematic method of drafting will enable the lawyer to communicate his intended meaning more effectively. That is the basic proposition to which this Article is addressed. This same proposition can be stated in a different form:

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1. The problem of interpretation is discussed in greater detail and the suggested approach is illustrated with respect to several sections of the Internal Revenue Code of 1954 in a forthcoming article.

2. A concise and clearly presented treatment of the six logical connectives examined here may be found in FITCH, *SYMBOLIC LOGIC: AN INTRODUCTION* 9-63 (1952).

If a more systematic method of drafting can be developed, then the lawyer will be able to communicate his intended meaning more effectively.

This proposition is itself a compound proposition made up of two subsidiary propositions:<sup>3</sup>

- P. a more systematic method of drafting can be developed
- Q. the lawyer will be able to communicate his intended meaning more effectively.

These subsidiary propositions are linked together by the words "if . . . then . . . ." to form the compound proposition, which in abbreviated form would be:

If P, then Q.

This "if . . . then . . . ." relationship between two propositions is called "implication," and is alternatively expressed as "P implies Q."<sup>4</sup>

In order to increase the clarity, precision and efficiency of thought, symbolic logicians represent relations such as implication by symbols.<sup>5</sup> Although there is complete freedom in selecting these symbols, effectiveness in thinking depends a great deal upon the system of notation that is used.<sup>6</sup> In this article a straight horizontal line "—————" will be

3. "Proposition" will be used here to refer to the intended meaning of a statement—its idea content.

4. The logician would be careful to distinguish four different kinds of implication:

1. Logical: If all men are mortal and Socrates is a man, then Socrates is mortal.
2. Definitional: If Mr. Black is a bachelor, then Mr. Black is unmarried.
3. Causal: If blue litmus paper is placed in acid, then the litmus paper will turn red.
4. Decisional: If Mr. Smith parks overtime, then Mr. Smith commits a traffic violation.

For the common core of meaning that is found in all four of these different kinds of implication, the logician has a special name: "material implication." COPI, *INTRODUCTION TO LOGIC* 229-36 (1953). Also, for some qualification of the use of "implies" to abbreviate "if . . . then . . . ." see QUINE, *MATHEMATICAL LOGIC* 27-30 (1951).

5. For those who would like to test this assertion, it is suggested that they attempt to solve the problem

$$742.96 \text{ divided by } 13.463$$

out to three decimal places, using words, not numbers, to perform the operations. The awkwardness of words as a means of describing the operations necessary to solve this problem becomes readily apparent. The mastery of numerical symbols as a means of manipulating quantitative relationships is a skill that everyone can recognize and appreciate. It is, perhaps, less generally known that the symbolic logicians achieve similar success in dealing with qualitative relationships.

6. Alfred North Whitehead, one of the foremost pioneers in symbolic logic, declares: "[B]y the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain." WHITEHEAD, *AN INTRODUCTION TO MATHEMATICS* 61 (1911). An illustration of what a difference nota-

used to represent implication.<sup>7</sup> For example, "P implies Q" can be represented:

- 1.1      1. P  


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           2. Q

The straight line will also represent implication when the compound proposition is written out in words:

- 1.2      1. A more systematic method of  
             drafting can be developed  


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           2. THE LAWYER WILL BE ABLE TO  
             COMMUNICATE HIS INTENDED MEANING  
             MORE EFFECTIVELY.

All statements that involve an implication can be expressed in this form, hereafter called the "systematically-pulverized" form.<sup>8</sup>

Most statements can be rearranged into the form of an implication without a change in the meaning of the statement. For example, the sentence:

All statements that involve an implication  
 can be expressed in systematically-pulverized form

is equivalent to:<sup>9</sup>

IF a statement involves an implication, THEN  
 such a statement can be expressed in systematically-  
 pulverized form.

And where:

P = a statement involves an implication  
 Q = such a statement can be expressed in  
 systematically-pulverized form,

this same statement can be abbreviated by the schematic:

tion can make is given by Copi. He points out how easy it is to multiply 113 by 9 compared with how difficult a task it is to multiply CXIII by IX. *COP*, *op. cit. supra* note 4, at 220. See also Jourdain, *The Nature of Mathematics*, in 1 *THE WORLD OF MATHEMATICS* 16 (Newman ed. 1956).

7. Strictly speaking from a logical viewpoint the straight line "—————" will represent material implication, the common core of meaning that is present in all the various kinds of implication. See note 4 *supra*. For our purposes here, however, we can consider "—————" as representing any of the different kinds of implication.

8. Why this name is appropriate will become apparent later. See p. 845 *infra*. Appreciation should be acknowledged to Professor Harold D. Lasswell, who first suggested calling it "creative-pulverization."

9. Symbolic logicians would use a universal quantifier to represent this:

(x) (IF x involves an implication, THEN x can be expressed in systematically-pulverized form).

However, because most readers will not be familiar with quantifier theory, it will be more convenient not to use quantifiers. The effect of the quantifiers will be achieved by the wording of the proposition.

- |     |    |   |
|-----|----|---|
| 1.3 | 1. | P |
|     | 2. | Q |

In systematically-pulverized form :

- |     |    |   |
|-----|----|---|
| 1.4 | 1. | A statement involves an implication   |
|     | 2. | SUCH A STATEMENT CAN BE<br>EXPRESSED IN SYSTEMATICALLY-<br>PULVERIZED FORM. |

The process of transforming an ordinary statement into systematically-pulverized form may be conveniently classified into four stages :

- A. pulverizing the statement into its constituent elements,
- B. rearranging the elements into approximately the form of an implication,
- C. discovering the appropriate schematic form,
- D. writing the statement in systematically-pulverized form.

A portion of section 397 of the *Restatement of Contracts* can serve to illustrate this :

“A breach . . . of a promise by one party to a bilateral contract, so material as to justify a refusal of the other party to perform a contractual duty, discharges that duty.”

A. Pulverize into constituent elements :

P = a breach of a promise by one party to a bilateral contract is so material as to justify a refusal of the other party to perform a contractual duty  
Q = such a breach discharges that duty

B. Rearrange into the form of an implication :

IF a breach of a promise by one party to a bilateral contract is so material as to justify a refusal of the other party to perform a contractual duty, THEN such a breach discharges that duty.

C. Discover the appropriate schematic form :

- |     |    |   |
|-----|----|---|
| 1.5 | 1. | P |
|     | 2. | Q |

D. Express in systematically-pulverized form :

- 1.6 1. A breach of a promise by one party to a bilateral contract is so material as to justify a refusal of the other party to perform a contractual duty.

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2. SUCH A BREACH DISCHARGES THAT DUTY.

The first of the two subsidiary propositions of an implication is called the "antecedent"; the second, the "consequent." The consequent Q results whenever the antecedent P prevails, or Q "follows" as a result of P. In this Article, in order to differentiate them, the antecedent is shown above the horizontal line and the consequent below. A final consequent is written in capital letters. In short:

1. antecedent
- 
2. CONSEQUENT.

2.0 *Conjunction*

Conjunction is the logical relationship between two subsidiary propositions that are joined by the idea expressed by the word "and" in a statement such as: "Roses are red AND violets are blue." In systematically-pulverized form conjunction is indicated by the symbol "&."<sup>10</sup> All propositions that are connected conjunctively will be enumerated in the following manner:<sup>11</sup>

1. P<sub>1</sub>  
 &2. P<sub>2</sub>  
 &3. P<sub>3</sub>  
 &4. P<sub>4</sub>

Conjunctive antecedents can imply a single consequent:

- 2.1 1. P<sub>1</sub>  
 &2. P<sub>2</sub>
- 
- 3 Q

A single antecedent can imply conjunctive consequents:

- 2.2 1. P
- 
2. Q<sub>1</sub>  
 &3. Q<sub>2</sub>

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10. Notice that this same idea is conveyed by many other English words, such as "but," "yet," "although," "however," "nevertheless," and "still." See CORI, *op. cit. supra* note 4, at 222-23.

11. This differs from the way two other connectives will be enumerated. See p. 847 *infra*.

And conjunctive antecedents can imply conjunctive consequents:

- |     |     |       |
|-----|-----|-------|
| 2.3 | 1.  | $P_1$ |
|     | &2. | $P_2$ |
|     |     |       |
|     | 3.  | $Q_1$ |
|     | &4. | $Q_2$ |

An example of a single antecedent implying conjunctive consequents, as in 2.2, is the following statement:

The consequent proposition of an implication is written in capital letters and is placed below the horizontal line.

This is equivalent to:

IF a proposition is a consequent of an implication, THEN that proposition is written in capital letters AND that proposition is placed below the horizontal line.

And in systematically-pulverized form:

- |     |     |   |
|-----|-----|---|
| 2.4 | 1.  | A proposition is a consequent of an implication                     |
|     |     |   |
|     | 2.  | THAT PROPOSITION IS WRITTEN IN CAPITAL LETTERS                      |
|     | &3. | THAT PROPOSITION IS PLACED BELOW THE HORIZONTAL LINE. <sup>12</sup> |

12. Notice that 2.4 can be condensed even further by avoiding the repetition of the words "THAT PROPOSITION IS" in the following manner:

- |     |    |   |
|-----|----|---|
| 2.4 | 1. | A proposition is a consequent of an implication |
|     |    |   |
|     | 2. | THAT PROPOSITION IS                             |
|     |    | 1. WRITTEN IN CAPITAL LETTERS                   |
|     |    | &2. PLACED BELOW THE HORIZONTAL LINE            |

In schematic form it would be:

- |     |    |           |
|-----|----|-----------|
| 2.2 | 1. | $P$       |
|     |    |           |
|     | 2. | .....     |
|     |    | 1. $Q_1$  |
|     |    | &2. $Q_2$ |

Hereafter, statements and schematic diagrams in systematically-pulverized form will be condensed in this manner. This means that the symbol "&" (and the other symbols as well) will be used to connect individuals and classes as well as propositions. Although this is a departure from the practice of the symbolic logician who would use different symbols, it should not create any difficulties because just what the items in such a connected list are can easily be ascertained by examining them.

The schematic 2.2 is equivalent to, and merely represents a more concise way of expressing, the pair of implications:

2.5	1.	$P_1$	&	2.6	1.	$P_1$
	2.	$Q_1$			2.	$Q_2$

Because it represents one pair of simple implications, and one pair only, a statement like 2.2 is still relatively specific and unambiguous even though it is more complex than a simple implication. However, notice that a statement with both conjunctive antecedents and conjunctive consequents like 2.3 does *not* have the specificity of more simple statements such as 2.1, 2.2, 2.5 and 2.6. A statement like 2.3 is more general, for it may represent any one pair among quite a few different pairs of implications. Thus, the statement

2.3	1.	$P_1$
	&2.	$P_2$
	3.	$Q_1$
	&4.	$Q_2$

may be used to represent any one pair from among no less than twenty different pairs of implications. For example, five of the twenty possible pairs would be:

1-	1.0	1. $P_1$	&	2.0	1. $P_1$	&2. $P_2$
		2. $Q_1$				3. $Q_2$
			OR			
2-	1.0	1. $P_1$	&	2.0	1. $P_2$	2. $Q_2$
		2. $Q_1$				2. $Q_2$
			OR			
3-	1.0	1. $P_1$	&	2.0	1. $P_2$	2. $Q_1$
		2. $Q_1$				&3. $Q_2$
			OR			
4-	1.0	1. $P_1$	&	2.0	1. $P_1$	&2. $P_2$
		2. $Q_2$				3. $Q_1$
			OR			
5-	1.0	1. $P_1$	&	2.0	1. $P_2$	2. $Q_1$
		2. $Q_2$				2. $Q_1$

Which pair a statement like 2.3 is intended to indicate cannot be determined until the context in which the statement appears is examined. Sometimes that context will indicate rather specifically just which pair is intended; other times the context will offer little guidance. The important thing for a draftsman to realize is that the generality (ambiguity?) of a statement tends to vary directly with its complexity, the addition of just a little complexity being accompanied by the possibility of a great deal of ambiguity.<sup>13</sup>

### 3.0 *Coimplication*

Coimplication can be defined as the conjunction of two particular implications—the coimplication of proposition P and proposition Q is the conjunction of the implication “P IMPLIES Q” and the implication “NOT P IMPLIES NOT Q.” Since the implication “NOT P IMPLIES NOT Q” is equivalent to the implication “Q IMPLIES P,” the coimplication “P COIMPLIES Q” can also be expressed as “P IMPLIES Q AND Q IMPLIES P.” Because coimplication is composed of two implications, it is appropriate to represent coimplication in systematically-pulverized form by two horizontal lines “                    .”

In schematic form, the coimplication

3.1 1. P <u>          </u> 2. Q	is equivalent to	3.2 1. P <u>          </u> 2. Q	&	3.3 1. NOT P <u>          </u> 2. NOT Q
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This, in turn, is the same as

3.1 1. P <u>          </u> 2. Q	is equivalent to	3.2 1. P <u>          </u> 2. Q	&	3.4 1. Q <u>          </u> 2. P
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Furthermore, it should be apparent that

3.1 1. P <u>          </u> 2. Q	is equivalent to	3.5 1. Q <u>          </u> 2. P
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13. The potential importance of this to the legal draftsman is apparent. Ordinarily when a draftsman wishes to express a statement that is broad and general in scope, he does so by his choice of words. He may use words like “reasonable” and “seasonable” to achieve generality and permit flexibility. Analysis of a statement like 2.3 shows rather vividly an alternate possibility for achieving generality where desired. Variation in the degree of generality expressed can be controlled by varying the complexity of the statement. Furthermore, it may be possible to exercise more sensitive control over the degree of generality of a statement if it is done by varying complexity, rather than by varying choice of words. A draftsman can clearly mark the limits that a statement is intended to cover by indicating which pair of subsidiary propositions it expresses. The boundaries of word categories cannot easily be so precisely defined. It may well be that the use of variation in complexity as a supplementary means of achieving generality will furnish a technique whereby the degree of generality of a statement can be more systematically and precisely controlled. It would seem to merit further inquiry.



An illustration of coimplication is easily constructed by making an addition to a statement made earlier. Recall the statement:

IF a more systematic method of drafting can be developed, THEN the lawyer will be able to communicate his intended meaning more effectively.

Add to this the implication:

IF a more systematic method of drafting can NOT be developed, THEN the lawyer will NOT be able to communicate his intended meaning more effectively,

and the two statements together form a coimplication. The first statement is expressed schematically in 3.2; the second, in 3.3. Together they form the coimplication in 3.1. Notice that another way of saying the second statement would be:

ONLY IF a more systematic method of drafting can be developed will the lawyer be able to communicate his intended meaning more effectively.

Hence, the two statements can be condensed into:

IF AND ONLY IF a more systematic method of drafting can be developed, THEN the lawyer will be able to communicate his intended meaning more effectively.

This, in turn, is equivalent to:

A more systematic method of drafting can be developed IF AND ONLY IF the lawyer will be able to communicate his intended meaning more effectively.

In systematically-pulverized form this would be:

- 3.6      1. A more systematic method of drafting can  
            be developed
- 
2. THE LAWYER WILL BE ABLE TO COMMUNICATE HIS INTENDED MEANING MORE EFFECTIVELY.

The following are equivalent ways of stating a coimplication like 3.1:

1. P COIMPLIES Q.
2. P IS EQUIVALENT TO Q.
3. P IMPLIES Q, AND Q IMPLIES P.
4. P IMPLIES Q, AND NOT P IMPLIES NOT Q.
5. IF P THEN Q, AND IF Q THEN P.
6. IF P THEN Q, AND IF NOT P THEN NOT Q.

In failing to make clear whether the relationship between two or more parts of a statement is intended to be implication or coimplication, legislatures frequently create an unnecessary problem of statutory construction. Many statutory provisions are in a form similar to: "Legal consequence Q will follow when the fact P is legally established." When rearranged, this forms the implication "IF P THEN Q." Courts faced with construing a provision such as this could well hold, as they often do, that it was the intent of the legislature to state only one of the factual antecedents that imply the legal consequence Q, *i.e.*, facts other than P could logically be held to imply Q.

But just as often courts apply the maxim "expressio unius est exclusio alterius," a rule of construction based on the assumption that in explicitly stating what antecedent implies consequence Q, legislatures intend that Q will follow *only* when the named fact P is established. In other words, the implication "IF P THEN Q" is converted into the coimplication "IF AND ONLY IF P THEN Q."

By expressing statutes in systematically-pulverized form, draftsmen would be reminded to indicate explicitly the intended meaning of the legislature. A statement of the form:

1.	P
2.	Q

would clearly indicate that an implication, and only an implication, was intended. On the other hand, a statement of the form:

1.	P
2.	Q

would clearly indicate that a coimplication was intended. Of course, in some situations the legislature intends to leave the relationship ambiguous, open to interpretation as either implication or coimplication. In those cases the form:

if P then Q

can be used to convey that meaning. Systematic pulverization will thereby help assure that any ambiguity of this type included in a statement is included intentionally and *not* inadvertently.

#### 4.0 *Exclusive Disjunction*

Another prevalent source of ambiguity is the logical relationship called "disjunction." The difficulty, to a large degree, is that there are two separate kinds of disjunction, and these are not always clearly distinguished. It is important to realize that there are these two possibilities open: a disjunctive statement is an *exclusive* disjunction or it is an *inclusive* disjunction.

An exclusive disjunction is a statement that asserts the truth of one or the other of its two subsidiary propositions, but not both. If it is assumed that no statements are both exclusively and inclusively disjunctive,<sup>14</sup> then an example of an exclusive disjunction is furnished in the last sentence in the previous paragraph:

- 4.1 A disjunctive statement is an exclusive disjunction, or it is an inclusive disjunction.

When rearranged into the form of an implication this statement becomes:

- 4.2 IF a statement is a disjunctive statement, THEN that statement is an exclusive disjunction OR it is an inclusive disjunction.

In schematic form this would be expressed:<sup>15</sup>

- 4.3 1. P  


---

 2. 1- Q<sub>1</sub>  
     OR  
     2- Q<sub>2</sub>

In systematically-pulverized form:

- 4.4 1. A statement is a disjunctive statement  


---

 2. THAT STATEMENT IS  
     1—AN EXCLUSIVE DISJUNCTION  
     OR  
     2—AN INCLUSIVE DISJUNCTION.

14. This assumption is used only for illustration and is actually false according to the customary definitions of exclusive and inclusive disjunction, *i.e.*, the following truth table definitions:

P	Q	P &OR Q	P OR Q
T	T	True	False
T	F	T	T
F	T	T	T
F	F	F	F

These definitions show that all statements that are exclusively disjunctive are also inclusively disjunctive. Therefore, it is false to assume that no statements are both inclusively and exclusively disjunctive.

15. Notice that the enumeration of the subsidiary propositions of an exclusive disjunction (1-, 2-, 3-, etc.) is different from the enumeration of the subsidiary propositions of a conjunction (1., 2., 3., etc.). The subsidiary propositions of a disjunction can be referred to as disjuncts. When the disjuncts are a single word or are very brief, it may be convenient

The symbol "OR" is used to indicate that the relationship between  $Q_1$  and  $Q_2$  is an exclusive disjunction. This symbol "OR" (and similarly "&OR" for *inclusive* disjunction) has been selected so as to be clearly distinct from the word "or," which is often used ambiguously to refer to both exclusive and inclusive disjunction. It will be convenient to retain this common ambiguous use of the word "or" in order to indicate a similar ambiguous relationship when the draftsman does not wish to state the kind of disjunction intended.

The statement 4.1 is a good example of the ambiguous use of the word "or." Often the context in which the word "or" appears will be helpful in determining whether the "or" is intended to indicate exclusive disjunction or inclusive disjunction. However, in this case the context is not very helpful. Unless the reader already has some notion of what is meant by "exclusive disjunction" and "inclusive disjunction," he does not know which 4.1 was intended to mean:

1- A disjunctive statement is an exclusive disjunction, or it is an inclusive disjunction, *but not both.*

OR

2- A disjunctive statement is an exclusive disjunction, or it is an inclusive disjunction, *or both.*

The intended meaning of the sentence is the meaning expressed by the second alternative, although, for purposes of illustrating exclusive disjunction, it was assumed that the statement meant the first alternative. In other contexts where the word "or" is used, its intended meaning is so clear from the context that it is unnecessary to express one of the end-phrases "but not both" or "or both." Thus, contexts vary in the extent to which they make clear the intended meaning of logical connectives between the constituent elements of a statement. The explicit symbols of systematic-pulverization help to clarify the intended meaning of such logical connectives in those contexts where the meaning intended would not otherwise be clear.

In summary, (1) a statement of the form "P or Q" is intended to represent an exclusive disjunction if, and only if, one, but not both, of its subsidiary propositions is intended; and (2) a statement involving exclusive disjunction will contain the symbol "OR" when expressed in systematically-pulverized form. The transformation of the first statement in this summary into systematically-pulverized form is of interest, for it illustrates rather vividly the appropriateness of the term "systematic pulverization." That statement is readily recognized as a complication with the schematic form:

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and may save space to omit the enumeration of the disjuncts. Thus, instead of being represented as in 4.3, an exclusive disjunction may be written:

$Q_1$  OR  $Q_2$

in schematic form and accompanied by a similar modification in systematically-pulverized form.

- 4.5      1.      P  
           —————  
           2.      Q

In systematically-pulverized form it is:

- 4.6      1. A statement of the form "P or Q" is  
           intended to represent an exclusively  
           disjunctive relationship  
           —————  
           2. ONE, BUT NOT BOTH, OF THAT STATEMENT'S  
           SUBSIDIARY PROPOSITIONS IS INTENDED.

Notice, however, that this coimplication statement can be further "pulverized" into simpler elements, for some further relationships are "hidden away" in the statement as expressed in 4.6. When proposition P is examined carefully, it is seen that P COIMPLIES Q can be separated into two subsidiary propositions, (1) P<sub>1</sub> and (2) P<sub>2</sub> COIMPLIES Q, which are joined by implication. Thus, 4.4 is equivalent in meaning to the implication:

- 4.7      IF a statement is of the form "P or Q,"  
           THEN such a statement is intended to represent  
           an exclusively disjunctive relationship IF AND  
           ONLY IF one, but NOT both, of that statement's  
           subsidiary propositions is intended.

The schematic form would be modified to become:

- 4.8      1.      P<sub>1</sub>  
           —————  
           2.      1.      P<sub>2</sub>  
                   —————  
                   2.      Q

Similarly, proposition Q can be separated into two subsidiary propositions, Q<sub>1</sub> and Q<sub>2</sub>, which are joined by conjunction. The statement:

One, but NOT both, of a statement's  
 subsidiary propositions is true

is equivalent in meaning to the conjunction:<sup>16</sup>

1. One of a statement's subsidiary  
 propositions is intended  
 &2. NOT both of a statement's subsidiary  
 propositions are intended.

16. This could be expressed in more condensed form as:

1.      1.      One  
           &2.    NOT both  
           of a statement's subsidiary propositions is true.

If Q is also pulverized into its constituent elements the schematic form must be further modified to become:

- 4.9      1.      P<sub>1</sub>
- 
- 2.      1.      P<sub>2</sub>
- 
- 2.      1.      Q<sub>1</sub>
- &2.      Q<sub>2</sub>
- .....

This can be read in a variety of ways. For example:

- 1- IF P<sub>1</sub>, THEN P<sub>2</sub> IF AND ONLY IF Q<sub>1</sub> AND Q<sub>2</sub>
- 2- IF P<sub>1</sub>, THEN P<sub>2</sub> COIMPLIES Q<sub>1</sub> AND Q<sub>2</sub>

In systematically-pulverized form the statement would be:

- 4.10      1. A statement is of the form "P or Q"
- 
- 2.      1. that statement is intended to  
            represent an exclusively disjunctive  
            relationship
- 
- 2.      1. ONE
- &2. NOT BOTH
- OF THAT STATEMENT'S SUBSIDIARY  
PROPOSITIONS IS INTENDED.

Just how far it will be appropriate to go in this process of pulverizing a statement into more simple elements must be decided by the draftsman with respect to each particular statement. At each stage further pulverization may or may *not* enable him to communicate his intended meaning more effectively. A draftsman must operate by intuition in arriving at what he thinks will be the optimum degree of pulverization. The second statement in the summary of exclusive disjunction illustrates this somewhat. When rearranged, that statement declares:

- 4.11      IF a statement involves an exclusive disjunction,  
            THEN that statement will contain the symbol "OR"  
            when expressed in systematically-pulverized form.

In systematically-pulverized form:

- 4.12      1. A statement involves an exclusive disjunction
- 
- 2. THAT STATEMENT WILL CONTAIN THE SYMBOL  
"OR" WHEN EXPRESSED IN SYSTEMATICALLY-  
PULVERIZED FORM.

It probably does not increase the communication of intended meaning to pulverize further the consequent of 4.12 and thus express the whole statement as:

- 4.13      1. A statement involves an exclusive disjunction
- 
2. 1. That statement is expressed in  
                    systematically-pulverized form
- 
2. THAT STATEMENT WILL CONTAIN THE  
                    SYMBOL "OR."

But this, of course, is a matter of judgment, to be exercised by the draftsman in each particular case.

5.0 *Inclusive Disjunction*

An inclusive disjunction is a statement that asserts that one or the other, or both, of its subsidiary propositions are true. The inclusively disjunctive relationship will be denoted in systematically-pulverized form by the symbol "&OR." The statement P &OR Q will mean:

P or Q or both,

and it will be systematically-pulverized as follows:

- 5.1      1)      P  
            2) &OR Q

The enumeration of inclusive disjunctions is thus distinguished from that of exclusive disjunctions and conjunctions:

CONJUNCTION	EXCLUSIVE DISJUNCTION	INCLUSIVE DISJUNCTION
1.    P &2.   Q	1-P OR 2-Q	1)    P 2) &OR Q

One additional observation should be made about disjunction. In 4.3 the disjunctive propositions were consequent propositions. It is also possible for either inclusive or exclusive disjunctions to appear as antecedent propositions, as in:

- 5.2      1.    1-    P<sub>1</sub>  
                    OR  
                    2-    P<sub>2</sub>
- 
2.    Q

- &5.3     1.     1)      $P_1$   
               2)     &OR  $P_2$
- 
2.     Q

In conclusion and as a convenient reminder, the definition of an inclusive disjunction will be expressed in systematically-pulverized form:

- 5.4     1. A statement is an inclusive disjunction
- 
2. THAT STATEMENT ASSERTS THAT
- 1- ONE  
                   OR  
               2- THE OTHER  
                   OR  
               3- BOTH
- OF ITS SUBSIDIARY PROPOSITIONS ARE  
           TRUE.

### 6.0 *Negation*

Negation will be discussed here in one short paragraph. The negate of this proposition is:

Negation will NOT be discussed here in one short paragraph.

Just as every positive number has a corresponding negative number, so every proposition has a negate. Negation is involved in a statement whenever the idea ordinarily expressed by the word "not" is present in that statement. For example, a negation is involved in a statement such as:

The article is NOT biased.

Such a statement will often be equivalent in meaning to another statement that does not explicitly contain a negation. For example:

The article is unbiased.

Whenever negation is expressed in systematically-pulverized form, it is generally preferable to use words like "NOT biased" rather than a word like "unbiased," in order to emphasize the presence of negation. In systematic pulverization the symbol "NOT" indicates negation.

### 7.0 *Relationship Among the Six Logical Connectives*

The six logical connectives described above are related to each other in a variety of ways.<sup>17</sup> For example, the following interrelationship of implica-

17. An understanding of this section is helpful but not essential to an understanding of systematic pulverization.



tion, conjunction, inclusive disjunction and negation can be logically demonstrated to be always true.<sup>18</sup>

7.1. (A)	1.	NOT	
	1.		P
	&2.		Q
	2.	1)	NOT P
		2)	&OR NOT Q

To test this assertion let:

P = the reader is tired

Q = the reader is bored.

Then the antecedent of 7.1 (A) states any one of the three following propositions:

- 1- The reader is NOT tired AND NOT bored  
OR
- 2- The reader is tired AND NOT bored  
OR
- 3- The reader is NOT tired AND bored.

It should be especially noted that a statement in the form of this antecedent definitely leaves open the possibility that either of the last two alternatives are the intended meaning of the statement. The consequent of 7.1 (A) states:

The reader is NOT tired &OR NOT bored.

The whole statement in systematically-pulverized form would be:

7.1 (A)	1.	The reader is NOT	
	1.	tired	
	&2.	bored	
	2.	THE READER IS	
	1)	NOT TIRED	
	2)	&OR NOT BORED.	

In ordinary prose the statement would assert the truism:

If the reader is either NOT tired AND NOT bored, OR is tired AND NOT bored, OR is NOT tired AND bored, THEN the reader is NOT tired AND/OR NOT bored.

---

18. Fitch shows how readily this is done in symbolic logic. Compare his highly efficient analysis in symbols with the illustration offered here in words. Representation in words is markedly clumsy and awkward by comparison. The equivalences shown in 7.1 through 7.8 are known as De Morgan's Theorem. FITCH, *op. cit. supra* note 2, at 60-62.

Similarly, the converse of 7.1(A) is also a truism. In schematic form it is:

7.1(B)	1.	1)	NOT P
		2)	&OR NOT Q
	2.	NOT	
		1.	P
		&2.	Q

The pair of implications 7.1(A) and 7.1(B) combine to form the co-implication:

7.1 (A&B)	1.	NOT	
		1.	P
		&2.	Q
	2.	1)	NOT P
		2)	&OR NOT Q

This indicates that the antecedent is equivalent to the consequent; an equivalent of the negation-conjunction proposition of the antecedent can be expressed by a disjunction-negation proposition.

In a similar manner it can be shown that an equivalent of the negation-conjunction antecedent can be expressed by an implication-negation proposition.<sup>19</sup> This would show that the following coimplication is also a truism:

7.1 (A&C)	1.	NOT	
		1.	P
		&2.	Q
	2.	1.	P
		2.	NOT Q

Finally, it can similarly be shown that the equivalent of a disjunction-negation antecedent can be expressed by an implication-negation proposition:

7.1 (B&C)	1.	1)	NOT P
		2)	&OR NOT Q
	2.	1.	P
		2.	NOT Q

---

19. In Fitch's system of logic the equivalence shown in 7.1 (A&C) can only be deduced when the principle of excluded middle is satisfied, *i.e.*, when the proposition is either true or not true. This condition will be satisfied in all of the situations where it is suggested that systematic pulverization be used.

In a similar manner the following interrelationships between implication, conjunction, negation and inclusive disjunction can be logically demonstrated to be always true.<sup>20</sup>

7.2 (A&B)

1. NOT  
     1. NOT P  
     &2. Q  


---

---

 2. 1) P  
     2) &OR NOT Q

7.2 (A&C)

1. NOT  
     1. NOT P  
     &2. Q  


---

---

 2. 1. NOT P  
     2. NOT Q

7.2 (B&C)

1. 1) P  
     2) &OR NOT Q  


---

---

 2. 1. NOT P  
     2. NOT Q

7.3 (A&B)

1. NOT  
     1. P  
     &2. NOT Q  


---

---

 2. 1) NOT P  
     2) &OR Q

7.3 (A&C)

1. NOT  
     1. P  
     &2. NOT Q  


---

---

 2. 1. P  
     2. Q

7.3 (B&C)

1. 1) NOT P  
     2) &OR Q  


---

---

 2. 1. P  
     2. Q

7.4 (A&B)

1. NOT  
     1. NOT P  
     &2. NOT Q  


---

---

 2. 1) P  
     2) &OR Q

7.4 (A&C)

1. NOT  
     1. NOT P  
     &2. NOT Q  


---

---

 2. 1. NOT P  
     2. Q

7.4 (B&C)

1. 1) P  
     2) &OR Q  


---

---

 2. 1. NOT P  
     2. Q

7.5 (A&B)

1. NOT P  
 &2. NOT Q  


---

---

 3. NOT  
     1) P  
     2) &OR Q

7.5 (A&C)

1. NOT P  
 &2. NOT Q  


---

---

 3. NOT  
     1. NOT P  
     2. Q

7.5 (B&C)

1. NOT  
     1) P  
     2) &OR Q  


---

---

 2. NOT  
     1. NOT P  
     2. Q

20. The reader can test these assertions by inserting hypothetical subsidiary propositions of his own.

<p>7.6 (A&amp;B)</p> <p>1. P</p> <p>&amp;2. NOT Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1) NOT P</p> <p style="padding-left: 2em;">2) &amp;OR Q</p>	<p>7.6 (A&amp;C)</p> <p>1. P</p> <p>&amp;2. NOT Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1. P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. Q</p>	<p>7.6 (B&amp;C)</p> <p>1. NOT</p> <p style="padding-left: 2em;">1) NOT P</p> <p style="padding-left: 2em;">2) &amp;OR Q</p> <hr style="border-top: 3px double #000;"/> <p>2. NOT</p> <p style="padding-left: 2em;">1. P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. Q</p>
<p>7.7 (A&amp;B)</p> <p>1. NOT P</p> <p>&amp;2. Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1) P</p> <p style="padding-left: 2em;">2) &amp;OR NOT Q</p>	<p>7.7 (A&amp;C)</p> <p>1. NOT P</p> <p>&amp;2. Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1. NOT P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. NOT Q</p>	<p>7.7 (B&amp;C)</p> <p>1. NOT</p> <p style="padding-left: 2em;">1) P</p> <p style="padding-left: 2em;">2) &amp;OR NOT Q</p> <hr style="border-top: 3px double #000;"/> <p>2. NOT</p> <p style="padding-left: 2em;">1. NOT P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. NOT Q</p>
<p>7.8 (A&amp;B)</p> <p>1. P</p> <p>&amp;2. Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1) NOT P</p> <p style="padding-left: 2em;">2) &amp;OR NOT Q</p>	<p>7.8 (A&amp;C)</p> <p>1. P</p> <p>&amp;2. Q</p> <hr style="border-top: 3px double #000;"/> <p>3. NOT</p> <p style="padding-left: 2em;">1. P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. NOT Q</p>	<p>7.8 (B&amp;C)</p> <p>1. NOT</p> <p style="padding-left: 2em;">1) NOT P</p> <p style="padding-left: 2em;">2) &amp;OR NOT Q</p> <hr style="border-top: 3px double #000;"/> <p>2. NOT</p> <p style="padding-left: 2em;">1. P</p> <hr style="border-top: 1px solid #000;"/> <p style="padding-left: 2em;">2. NOT Q</p>

This set of schematics indicates the relationships among five of the logical connectives: implication, coimplication, conjunction, negation and inclusive disjunction. The other connective, exclusive disjunction, can be shown to be equivalent to an expression involving negation and coimplication:

<p>7.9</p> <p>1. 1- P</p> <p style="padding-left: 2em;">OR</p> <p style="padding-left: 2em;">2- Q</p> <hr style="border-top: 3px double #000;"/> <p>2. 1. NOT P</p> <hr style="border-top: 3px double #000;"/> <p style="padding-left: 2em;">2. Q</p>
---

Expression in systematically-pulverized form also furnishes a conveniently brief, but at the same time comprehensive, way to distinguish between inclusive disjunction and exclusive disjunction.

## 7.10 Inclusive Disjunction

1. P &OR Q
2. 1- P & NOT Q  
OR  
2- NOT P & Q  
OR  
3- P & Q
- &3. NOT  
1. NOT P  
&2. NOT Q

## 7.11 Exclusive Disjunction

1. P OR Q
2. 1- P & NOT Q  
OR  
2- NOT P & Q
- &3. NOT  
1. NOT P  
&2. NOT Q
- &4. NOT  
1. P  
&2. Q

Because the process of systematic-pulverization presented here is an attempt to strike a workable compromise between:

1. symbolic logic
- &2. statements in the ordinary language of the practicing lawyers

one of the guiding aims in formulating the systematically-pulverized form has been to steer clear of unfamiliar symbols wherever possible. A reader does not need to have a flair for mathematics to understand systematic-pulverization. The symbol for implication "—————" is the only one that will be new to most readers. The symbols for the other connectives, "&," "&OR," "OR" and "NOT," are already somewhat familiar; and "=====" can readily be derived from "—————."

8.0 *Summary*

Before turning to some illustrative applications of systematic pulverization to concrete legal problems in the second section, it will be useful to have available a summary of the six logical connectives.

CONNECTIVE	SYMBOL	EXAMPLES	
		ORDINARY VERBAL FORM	SYSTEMATICALLY-PULVERIZED FORM
Conjunction	1. &2.	The six logical connectives dealt with here are conjunction, exclusive disjunction, inclusive disjunction, negation, implication and coimplication.	The six logical connectives dealt with here are 1. conjunction &2. exclusive disjunction &3. inclusive disjunction &4. negation &5. implication &6. coimplication.
Exclusive disjunction	1- OR 2-	A person either understands them or he does not.	A person either 1- does OR 2- does NOT understand them
Inclusive disjunction	1) 2) &OR	Exclusive disjunction and/or inclusive disjunction may prove tricky for a while, but one soon learns to distinguish them.	1) Exclusive disjunction 2) &OR inclusive disjunction may prove tricky for a while, but one soon learns to distinguish them.
Negation	NOT	The explanation here should not be hard to understand.	The explanation here should NOT be hard to understand.
Implication	1. _____ 2.	If a person can read, then he should be able to understand it very easily.	1. A person can read _____ 2. HE SHOULD BE ABLE TO UNDERSTAND IT VERY EASILY.
Coimplication	1. _____ 2.	If, and only if, a person can read, he should be able to understand it very easily.	1. A person can read _____ 2. HE SHOULD BE ABLE TO UNDERSTAND IT VERY EASILY
<p style="text-align: center;">1. Antecedent _____ 2. CONSEQUENT</p>			

## APPLICATION TO CONCRETE LEGAL PROBLEMS

By using systematic pulverization a draftsman can more exactly express his intended meaning, so that those who must interpret and apply the instrument need not speculate as much about probable intention. At the same time, the draftsman will be alerted against the inadvertent inclusion of ambiguity, which may lead to unnecessary litigation. Furthermore, in the interpretation of instruments drafted in the traditional manner, systematic pulverization can be used to discover the wide variety of possible interpretations that are logically available. Both of these uses—drafting and interpretation—can be demonstrated by reference to specific ambiguities in loosely drafted legal instruments.

9.0 *Implication-Coimplication Ambiguity*

An illustration of one of the most commonly overlooked ambiguities—whether the connection between two elements of a statement is intended to be implication or coimplication—is found in section 65 of the Uniform Sales Act:

## “Sec. 65 WHEN SELLER MAY RESCIND CONTRACT OR SALE

Where the goods have not been delivered to the buyer, and the buyer has repudiated the contract to sell or sale, or has manifested his inability to perform his obligations thereunder, or has committed a material breach thereof, the seller may totally rescind the contract or the sale by giving notice of his election so to do to the buyer.”

The essential idea in systematically-pulverizing a proposition like section 65 is to separate the statement into its constituent elements and then to determine the appropriate logical relationships between them.<sup>21</sup> One convenient breakdown of section 65 is the following:

- A = Where the goods have NOT been delivered to the buyer
- B = the buyer has repudiated the contract to sell OR the sale
- C = the buyer has manifested his inability to perform his obligations thereunder
- D = the buyer has committed a material breach thereof
- E = the seller gives notice of his election to rescind to the buyer
- F = THE SELLER MAY TOTALLY RESCIND THE CONTRACT OR SALE.

The context indicates that the ambiguous “or” connecting B, C and D is an inclusive disjunction so that on its face the statute says:

- |     |     |   |     |   |     |   |
|-----|-----|---|-----|---|-----|---|
| 9.1 | 1.  | A |     |   |     |   |
|     | &2. | B | &OR | C | &OR | D |
|     | &3. | E |     |   |     |   |
|     |     |   |     |   |     |   |
|     | 4.  | F |     |   |     |   |

21. This process works in a similar manner whether used to interpret a given statement or to draft a new one.

Section 65 clearly declares that if the other antecedents are satisfied then:

IF (E) the seller notifies,  
THEN (F) the seller may rescind.

But does "expressio unius est exclusio alterius" apply to notification?<sup>22</sup> If the other antecedents are satisfied, must the seller notify the buyer of his intention before he can rescind, or are there other pathways open for the seller to gain the right to rescind in addition to the one explicitly expressed in section 65? If the expression of a seller's right to rescind by way of notification is intended to exclude all other possibilities, then section 65 would be interpreted as follows:

9.2	1.	A				
	&2.	B	&OR	C	&OR	D
	3.	1.	<u>E</u>			
		2.	<u>F</u>			

If the draftsman had been using systematic pulverization the question of whether 9.1 or 9.2 was intended would have been brought to his attention; he would have been reminded to indicate his choice between them, if he desired to do so. On its face section 65 does not indicate clearly which of these two interpretations was intended. For that matter, there are six other possible ways of interpreting the logical relationships between F and the other elements:

9.3	1.	A					
	&2.	E					
	3.	1.	B	&OR	C	&OR	D
			<u>F</u>				
		2.	<u>F</u>				
9.4	1.	B	&OR	C	&OR	D	
	&2.	E					
	3.	1.	<u>A</u>				
		2.	<u>F</u>				
9.5	1.	A					
	2.	1.	B	&OR	C	&OR	D
		&2.	<u>E</u>				
		3.	<u>F</u>				

22. The alternative route to a rescission right for the seller indicated in § 61 of the Uniform Sales Act suggests that the latin maxim should not be applied to notification.



9.6	1.	B	&OR	C	&OR	D
	2.	1. A				
		&2. E				
		3. F				
9.7	1.	E				
	2.	1. A				
		&2. B	&OR	C	&OR	D
		3. F				
9.8	1.	A				
	&2.	B	&OR	C	&OR	D
	&3.	E				
	4.	F				

For a given statement the number of possible implication-coimplication interpretations of the statement can be mathematically determined. Where the number of antecedents in the statement =  $N$ , the number of possible interpretations =  $2^N$ . In this case  $N = 3$ , so the number of possible interpretations =  $2^3 = 8$ .

Thus, for what appears to be a relatively simple and straightforward statutory passage, there are often a wide variety of possible interpretations. In section 65 there are eight different combinations of implication and coimplication for a court to choose among. It is suggested that in many—but certainly not all—such cases the consensus of the legislature would be embodied in just one of the possible interpretations, and that ought to be specified clearly, rather than expressed in the usual broad and ambiguous form. This example illustrates how systematic pulverization, by the questions it raises, can be used as a tool to lead the legislature to express more clearly just what it does intend—at least in those cases where it wishes to express a clear intention. It also illustrates the usefulness of systematic pulverization for the advocate, who is provided with a comprehensive and systematic reminder of all the possible interpretations he might argue for his client.<sup>23</sup>

When section 65 is read in the light of section 53 (*Remedies of an Unpaid Seller*) and section 61 (*When and how the Seller May Rescind the Sale*), the

23. Only the implication-coimplication ambiguity of § 65 has been considered here. But it should also be apparent that all of the uses of the word "or" in the section are ambiguous, and that the appropriate interpretation for each instance of its usage must be determined.

most reasonable interpretation of section 65 would seem to be 9.1. In systematically-pulverized form this would be:

- 9.1      1. The goods have NOT been delivered to the buyer  
           &2. The buyer has  
               1)            repudiated the contract to sell OR sale  
               2) &OR manifested his inability to perform  
                               his obligations thereunder  
               3) &OR committed a material breach thereof  
           &3 the seller gives notice of his election to rescind  
               to the buyer

---

4. THE SELLER MAY TOTALLY RESCIND THE CONTRACT OR THE SALE.

Courts are often faced with resolving ambiguities as to whether implication or complication is intended in a statement.<sup>24</sup> It is likely that systematic pulverization will help avoid some of the litigation built upon such ambiguity. Resolving such ambiguity is not always an easy task, and judicial responses do not follow a uniform rule in resolving it.<sup>25</sup>

#### 10.0 *Disjunctive-Conjunctive Ambiguity*

In the construction of wills, courts that are called upon to give specific meaning to the words "and" and "or" frequently substitute an "or" for an "and," and vice versa, in order to achieve the apparent intent of the testator.<sup>26</sup> But often the "plain meaning" of the words is held to foreclose any further inquiry into the testator's intent. A classic instance of such strict interpretation occurred in *Chichester Diocesan Fund v. Simpson*,<sup>27</sup> in which the House of Lords was called upon to construe a residuary bequest to testator's executors in trust

"for such charitable institution or institutions or other charitable or benevolent object or objects . . . as [his] . . . executors . . . may in their . . . absolute discretion select. . . ."

The important ambiguity was the word "or" used to connect the words "charitable" and "benevolent." It is a cardinal rule of English common law that a man can not delegate his testamentary power. Lord Simonds explained that there is only one exception to this rule:

---

24. See cases in DEC. DIG. *Statutes* key no. 195, "Express mention and implied exclusion."

25. Three recent cases in Illinois illustrate how courts find it appropriate to vacillate between application and rejection of the implied exclusion rule. See *Dick v. Roberts*, 8 Ill. 2d 215, 133 N.E.2d 305 (1956); *In re Leichtenberg's Estate*, 7 Ill. 2d 545, 131 N.E.2d 487 (1956); *Dilton v. Nathan*, 10 Ill. App. 2d 289, 135 N.E.2d 136 (1956).

26. See cases in DEC. DIG. *Statutes* key no. 197, "Conjunctive and disjunctive words."

27. [1944] A.C. 341.

"A testator may validly leave it to his executors to determine what charitable objects shall benefit, so long as charitable and no other objects may benefit."<sup>28</sup>

Since the "or" was interpreted by the majority of the Lords to indicate disjunction, the executors under the will would have been empowered to distribute to objects that were benevolent but *not* charitable, and the will was thus held invalid. The rigidity of interpretation insisted upon in this decision may seem undesirable to readers accustomed to the more flexible spirit of most courts. Although such flexibility is clearly desirable to permit the achievement of justice in each particular case, it does enhance uncertainty, which may, in turn, encourage litigation.<sup>29</sup> Interpretations of the "or" as either conjunction (charitable AND benevolent) or as coimplication (charitable THAT IS TO SAY benevolent) would have saved the will, because either of these interpretations would require every distribution by the executor to be charitable.

This was clearly a will that would have been saved if the draftsman had been using systematic pulverization. When he came to the troublesome "or" between "charitable" and "benevolent," he would have been faced with a specific choice in systematically pulverizing. He would have been forced to make a decision to represent that "or" by one of the following five symbols:

- 1- "&" indicating conjunction,
- 2- "====" indicating coimplication,
- 3- "&OR" indicating inclusive disjunction,
- 4- "OR" indicating exclusive disjunction,
- 5- "or" indicating that the draftsman wished to be ambiguous.

There is little doubt that if the draftsman had been faced with this choice, a valid will would have been written by his specifying one of the first two choices.<sup>30</sup> In this fashion systematic pulverization provides the draftsman with a reminder to scrutinize small but significant details more thoroughly.

28. *Id.* at 371.

29. In wills, for example, just how "and" or "or" will be interpreted is difficult to predict. See 95 C.J.S. *Wills* § 613(b) (1957).

30. Goddard, L.J., voting to invalidate the will in the Court of Appeal, observed:

"For myself I cannot have any doubt that the draughtsman in this case fell into a trap, because it is obvious that the [testator's] intention was to leave the money to charity in the popular sense of the term, and, had it been pointed out to him when he said, 'I want to leave it to charitable or benevolent objects,' 'well, if you use those words the money will not go to charity but to your first cousins once removed' (of whose existence he himself probably did not know) then, provided [that the testator] was of sound mind and memory and understanding, there is not the least doubt . . . that he would have said, 'Cut out the word "benevolent".' "

*In re Diplock*, [1941] 1 Ch. 253, 267 (C.A.).

### 11.0 *Ambiguity of Reference*

In addition to the ambiguity involved in interpreting which logical connectives are intended, a second kind, ambiguity of reference, can also be minimized by systematic pulverization. Statements that contain this kind of ambiguity are called "amphibolous" by symbolic logicians.<sup>31</sup> The ambiguity arises out of a loose combination of words such that it is not clear which word or phrase refers to which other word or phrase. A common arithmetical example would be:

$$4 + 3 \times 2 = ?$$

Which is it: 10 or 14? In arithmetic such ambiguity can be clarified by the use of parentheses:

$$4 + (3 \times 2) = 10$$

$$(4 + 3) \times 2 = 14$$

Systematic pulverization can clarify amphibolous statements in words in a manner similar to the way that parentheses clarify amphibolous arithmetical statements. For example, take the statement:

All law professors and students at Yale should have little trouble understanding this.

Who is it that should have little trouble understanding this? Is it

1- all law professors (throughout the world) and  
(law) students at Yale

OR

2- all law professors (at Yale) and  
(law) students at Yale

OR

3- all law professors (throughout the world) and  
(all kinds of) students at Yale

OR

4- all law professors (at Yale) and  
(all kinds of) students at Yale?

The statement is amphibolous because it does not clearly state which of the four propositions is meant. However, in systematically-pulverized form the meaning is indicated clearly by explicit changes in the form of the antecedent. The first proposition would be:

1. A person is a law
  - 1) professor
  - 2) &OR student at Yale

- 
2. THAT PERSON SHOULD HAVE LITTLE TROUBLE UNDERSTANDING THIS.
- 

31. See CORI, *op. cit. supra* note 4, at 70.

The second proposition:

1. A person is a law
  - 1) professor
  - 2) &OR student
 at Yale
 

---
2. THAT PERSON SHOULD HAVE LITTLE TROUBLE UNDERSTANDING THIS.

The third proposition:

1. A person is a
  - 1) law professor
  - 2) &OR student at Yale

---
2. THAT PERSON SHOULD HAVE LITTLE TROUBLE UNDERSTANDING THIS.

The fourth proposition:

1. A person is a
  - 1) law professor
  - 2) &OR student
 at Yale
 

---
2. THAT PERSON SHOULD HAVE LITTLE TROUBLE UNDERSTANDING THIS.

A concrete example of this kind of ambiguity in a legal context is found in section 53 of the Uniform Sales Act:

*"Sec. 53 Remedies of an Unpaid Seller*

- (1) Subject to the provisions of this act, notwithstanding that the property in the goods may have passed to the buyer, the unpaid seller of goods, as such, has—
  - (a) a lien on the goods or right to retain them for the price while he is in possession of them. . . ."

Suppose an unpaid seller of the goods does not have possession of them. Does he have a lien on the goods? In other words, does the phrase "while he is in possession of them" modify

- 1— both
  - A. "right to retain them for the price"
  - &B. "lien on the goods"
 OR
- 2— only "right to retain them for a price"?

The probable intent is that it should modify both. In systematically-pulverized form this would be clearly apparent:

- 11.1 1. There is an absence of provisions otherwise in this act  
 &2. whether OR NOT property in the goods has passed to the buyer <sup>32</sup>  
 &3. a person is an unpaid seller of goods
- 
4. THAT PERSON HAS A  
 1) LIEN ON THE GOODS  
 2) &OR RIGHT TO RETAIN THEM FOR THE PRICE  
 WHILE HE IS IN POSSESSION OF THEM.

If the phrase were intended to modify only "right to retain them," then it would appear:

- 11.2 1. ....  
 &2. ....  
 &3. ....
- 
4. ....  
 1) .....  
 2) &OR RIGHT TO RETAIN THEM FOR THE PRICE WHILE HE IS IN POSSESSION OF THEM.

Closer scrutiny will reveal that there is a complication in the consequent of section 53 that is uncovered if the consequent is pulverized further. If the phrase "while he is in possession of them" is intended to modify both "lien on the goods" and "right to retain them," then the further pulverized consequent would be:

- 11.3 1. ....  
 &2. ....  
 &3. ....
- 
4. 1. that person is in possession of the goods  
 2. THAT PERSON HAS A  
 1) LIEN ON THE GOODS  
 2) &OR RIGHT TO RETAIN THEM FOR THE PRICE

---

32. Note that this antecedent does not have any logical relevance. It was in the original statement for purposes of emphasis and serves only that same purpose here. It could be omitted entirely without changing the meaning of the statement.

On the other hand, if it were intended that the phrase "while he is in possession of them" should modify only "right to retain them," then the further pulverized consequent would be:

- 11.4 1. ....
- &2. ....
- &3. ....

---

4. THAT PERSON

- 1) HAS A LIEN ON THE GOODS
- 2) &OR 1. is in possession of the goods

---

2. HAS A RIGHT TO RETAIN THEM FOR THE PRICE.

A second example illustrating ambiguity of reference in a concrete legal context is found in section 1448 of the New York Civil Practice Act. The relevant portion of that section states:

"Section 1448 *Validity of Arbitration Contracts or Submissions*  
 "... [a] provision in a written contract between a labor organization . . . and employer . . . to settle by arbitration a controversy or controversies . . . thereafter arising between the parties . . . shall . . . be valid. . . ."

Does this section apply to labor-employer agreements for arbitration of

- 1- future disputes only
- OR
- 2- both future disputes and present disputes?

As presently written it could be interpreted to apply to either. If the section were expressed in systematically-pulverized form, the structure of the antecedent could remove all doubt. If it was intended to apply to arbitration agreements for both future and present disputes, the section would read:

- 11.5 1. A provision is in a written contract between a labor organization & employer to settle by arbitration
    - 1) a controversy
    - 2) &OR controversies thereafter arising between the parties
- 
- 2. THAT PROVISION SHALL BE VALID.

However, if it was intended to apply to future disputes only, it would read:

- 11.6 1. ....
    - 1) a controversy
    - 2) &OR controversies thereafter arising between the parties
- 
- 2. ....

## 12.0 Comparison

One other way in which systematic pulverization will be useful is for comparing statements, so that differences and similarities can be pinpointed. This use can be illustrated by comparing the M'Naghten Rule with a proposed replacement suggested by Professor Jerome Hall in a recent article.<sup>33</sup> This particular example is fruitful for another reason also. It shows how an eminent authority (such as Jerome Hall in criminal law) who is carefully drafting a statement to express just exactly what he means about something in his own special field can easily fall into an error of omission when expressing his thoughts in ordinary prose. If the rule were expressed in systematically-pulverized form such an error would be made sufficiently glaring that the possibility of its occurrence would be virtually eliminated.

In ordinary prose the two rules to be compared are expressed as follows:

### *The M'Naghten Rule*

"The Jury ought to be told in all cases . . . that to establish a defense on the ground of insanity, it must be clearly proved that, at the time of the committing of the act, the party accused was laboring under such a defect of reason, from disease of the mind, as not to know the nature and quality of the act he was doing, or if he did know it, that he did not know he was doing what was wrong."<sup>34</sup>

### *Hall's Suggested Rule*

"A crime is not committed by anyone who, because of a mental disease, is unable to understand what he is doing and to control his conduct at the time he commits a harm forbidden by criminal law. In deciding this question with reference to the criminal conduct with which the defendant is charged, the trier of facts should decide (1) whether because of mental disease, the defendant lacked the capacity to understand the physical nature and consequences of his conduct; and (2) whether, because of such disease, the defendant lacked the capacity to realize that it was morally wrong to commit the harm in question."<sup>35</sup>

A convenient pulverization of the relevant portions of the M'Naghten Rule into its component elements would be:

- A<sub>1</sub> = It is clearly proved that the accused party was laboring under such a defect of reason *as NOT to know the nature and quality of the act he was doing* at the time of committing the act
- B<sub>1</sub> = It is clearly proved that the accused party was laboring under such a defect of reason *that he did NOT know he was doing what was wrong* at the time of committing the act
- C<sub>1</sub> = It is clearly proved that *the defect of reason is from disease of the mind*
- D<sub>1</sub> = A DEFENSE IS ESTABLISHED ON THE GROUND OF INSANITY.

33. Hall, *Psychiatry and Criminal Responsibility*, 65 YALE L.J. 761 (1956).

34. M'Naghten's Case, 10 Cl. & Fin. 200, 210, 8 Eng. Rep. 718, 722 (H.L. 1843).

35. Hall, *supra* note 33, at 781.



In schematic form the M'Naghten Rule would be:

- 12.1 1. ....  
       1. ....  
           1)       A<sub>1</sub>  
           2) &OR B<sub>1</sub>  
           .....  
       &2.       C<sub>1</sub>  
       -----  
       2.       D<sub>1</sub>

In systematically-pulverized form:

- 12.2 1. It is clearly proved that  
       1. the accused party was laboring under such  
           a defect of reason  
           1)       as NOT to know the nature  
                   & quality of the act he was  
                   doing  
           2) &OR that he did NOT know  
                   that he was doing what  
                   was wrong  
       &2. at the time of committing the act  
           this defect of reason is from  
           disease of the mind

-----  
 2. A DEFENSE IS ESTABLISHED ON THE GROUND  
 OF INSANITY.

The Hall Rule is somewhat more complex. It ultimately reaches a similar result (or seemingly is intended to), but with modernized language and in two stages. A convenient pulverization into component elements would be:

- A<sub>2</sub> = The trier of facts decides that the defendant lacked the capacity to *understand the physical nature and consequences of his conduct* at the time he committed a harm forbidden by the criminal law  
 B<sub>2</sub> = The trier of facts decides that the defendant lacked the capacity to *realize that it was morally wrong to commit the harm in question* at the time he committed a harm forbidden by criminal law  
 C<sub>2</sub> = The trier of facts decides that *this lack of capacity is because of a mental disease*  
 E = the defendant is unable to *understand what he is doing* at the time he commits a harm forbidden by criminal law  
 F = the defendant is unable to *control his conduct* at the time he commits a harm forbidden by criminal law  
 G = the defendant's inability to understand what he is doing & to control his conduct is because of a mental disease  
 D<sub>2</sub> = A CRIME CANNOT BE COMMITTED BY THAT DEFENDANT.

The first part of the Hall Rule in schematic form would be:

- 12.3 1. ....
  - 1. E
  - &2. F
  - .....
- &2. G

---

- 3. D<sub>2</sub>

In systematically-pulverized form this would be:<sup>36</sup>

- 12.4 1. The defendant is unable to
  - 1. understand what he is doing
  - &2. control his conduct
 at the time he commits a harm forbidden by criminal law
- &2. the defendant's inability to understand what he is doing & to control his conduct is because of a mental disease

---

- 3. A CRIME CANNOT BE COMMITTED BY THAT DEFENDANT

The second part of the Hall Rule is a little more tricky. The phrase, "in deciding this question," apparently refers to whether E, F and G are satisfied. Assuming this reference is so intended, then the second part of the Hall Rule would be:

- 12.5 1. ....
  - 1. ....
    - 1. E
    - &2. F
    - .....
  - &2. G

---

- 2. ....
  - 1. ....
    - 1. A<sub>2</sub>
    - &2. B<sub>2</sub>
    - .....
  - &2. C<sub>2</sub>

36. Although Hall does not explicitly so provide in the language used, he probably intends this statement to be a coimplication, rather than merely an implication as shown.

In systematically-pulverized form:

- 12.6 1. The trier of facts is to decide whether
1. the defendant is unable to
    1. understand what he is doing
    - &2. control his conduct
 at the time he commits a harm forbidden by criminal law
  - &2. the defendant's inability to understand what he is doing & to control his conduct is because of a mental disease

---

2. THE TRIER OF FACTS SHOULD DECIDE WHETHER

1. THE DEFENDANT LACKED THE CAPACITY TO
  1. UNDERSTAND THE PHYSICAL NATURE & CONSEQUENCES OF HIS CONDUCT
  - &2. REALIZE THAT IT WAS MORALLY WRONG TO COMMIT THE HARM IN QUESTION
 AT THE TIME HE COMMITTED A HARM FORBIDDEN BY CRIMINAL LAW
- &2. THIS LACK OF CAPACITY IS BECAUSE OF A MENTAL DISEASE.

But when the trier of facts reaches decisions as to  $A_2$ ,  $B_2$ , and  $C_2$ , then what? When the Hall Rule is systematically-pulverized, it becomes apparent that its first and second parts go to the threshold of a modernized version of the M'Naghten Rule, but that it does *not* explicitly state a complete rule.

For example, suppose the trier of facts decides:

1. NOT  $A_2$  the defendant did NOT lack the capacity to *understand the physical nature and consequences of his conduct* at the time he committed a harm forbidden by criminal law
- &2.  $B_2$  the defendant lacked the capacity to *realize that it was morally wrong to commit the harm in question* at the time he committed a harm forbidden by criminal law.

From such a set of decisions, to what conclusions does the Hall Rule lead in regard to whether:

1.       E    the defendant is unable to *understand what he is doing* at the time he commits a harm forbidden by criminal law
- &2.      F    the defendant is unable to *control his conduct* at the time he commits a harm forbidden by criminal law?

Or suppose that the trier of facts decides  $A_2$  and NOT  $B_2$ . Applying the Hall Rule, what would this indicate about E and F? Or suppose that the trier of facts decides

$A_2$  and  $B_2$   
OR  
NOT  $A_2$  and NOT  $B_2$ ,

what does either of these indicate about E and F? It is suggested that 12.4 and 12.6 (the Hall Rule) do not expressly indicate anything about E and F as a result of findings by the trier of facts on  $A_2$  and  $B_2$ . The Hall Rule does not explicitly state what logical connection there is between:

1. the findings of the trier of facts on whether the defendant lacked the capacity to
  - $A_2$  understand the physical nature & consequences of his conduct
  - & $B_2$  realize that it was morally wrong to commit the harm in question

and whether

2. the defendant is unable to
  - E understand what he is doing
  - &F control his conduct.

For example, does either  $A_2$  or  $B_2$  alone imply E and F, or are both required before E and F follow? The unstated connection must be read in by the reader or trier of facts. Apparently the unarticulated proposition that Professor Hall had in mind is the following:<sup>37</sup>

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37. This is not necessarily the unarticulated proposition that Professor Hall had in mind. For example, the logical relationship between  $A_2$  and  $B_2$  may be "&" or "OR," instead of "&OR" as shown. It is suggested, however, that 12.7 is the most probable version of the unarticulated proposition.

- 12.7    1. ....  
           1. ....  
               1)           A<sub>2</sub>  
               2) &OR B<sub>2</sub>  
                   .....  
           &2.       C<sub>2</sub>
- 
2. ....  
               1.    E  
               &2.   F  
                   .....  
           &3.   G

In systematically-pulverized form:

- 12.8    1. The trier of facts decides that  
           1. the defendant lacked the capacity to  
               1)       understand the physical nature  
                       & consequences of his conduct  
               2) &OR realize that it was morally wrong  
                       to commit the harm in question  
               at the time he committed a harm  
               forbidden by criminal law  
           &2. this lack of capacity is because of  
               a mental disease
- 
2. THE DEFENDANT IS UNABLE TO  
               1. UNDERSTAND WHAT HE IS DOING  
               &2. CONTROL HIS CONDUCT  
               AT THE TIME HE COMMITS A HARM FORBID-  
               DEN BY CRIMINAL LAW  
           &3. THE DEFENDANT'S INABILITY TO UNDER-  
               STAND WHAT HE IS DOING & TO CONTROL  
               HIS CONDUCT IS BECAUSE OF A MENTAL  
               DISEASE.

If it is assumed that 12.4 is intended to be a coimplication (as it probably is), then given the propositions 12.4 and 12.8, it is possible to infer the proposition:<sup>38</sup>

---

38. The logical proof of this inference is a relatively simple one and is of the form:  
 IF P COIMPLIES Q (12.8) and Q COIMPLIES R (12.4),  
 THEN P COIMPLIES R (12.9).

- 12.9 1. ....
  - 1. ....
    - 1) A<sub>2</sub>
    - 2) &OR B<sub>2</sub>
  - .....
  - &2. C<sub>2</sub>

---

- 2. D<sub>2</sub>

In systematically-pulverized form this would be :

- 12.10 1. The trier of facts decides that
  - 1. the defendant lacked the capacity to
    - 1) understand the physical nature & consequences of his conduct
    - 2) &OR realize that it was morally wrong to commit the harm in question
  - at the time he committed a harm forbidden by the criminal law
  - &2. this lack of capacity is because of a mental disease

---

2. A CRIME CANNOT BE COMMITTED BY THAT DEFENDANT.

These transformations have at last put the Hall Rule in a form that can readily be compared with the M'Naghten Rule. Notice the similarity of their schematic representations :

- 12.1 1. ....
  - 1. ....
    - 1) A<sub>1</sub>
    - 2) &OR B<sub>1</sub>
  - .....
  - &2. C<sub>1</sub>

---

- 2. D<sub>1</sub>

- 12.9 1. ....
  - 1. ....
    - 1) A<sub>2</sub>
    - 2) &OR B<sub>2</sub>
  - .....
  - &2. C<sub>2</sub>

---

- 2. D<sub>2</sub>

The comparison of the two rules in systematically-pulverized form would be :

12.2 THE M'NAGHTEN RULE

12.10 The HALL RULE

- 1. It is clearly proved that
  - 1. the accused party was laboring under such a defect of reason
    - 1) as NOT to know the
      - 1. nature
      - &2. quality of the act he was doing
    - 2) &OR that he did NOT know he was doing what was wrong
- at the time of committing the act
- &2. this defect of reason is from disease of the mind

- 1. The trier of facts decides that
  - 1. the defendant lacked the capacity to
    - 1) understand the
      - 1. physical nature
      - &2. consequences of his conduct
    - 2) &OR realize that it was morally wrong to commit the harm in question
- at the time he committed a harm forbidden by the criminal law
- &2. this lack of capacity is because of a mental disease

2. A DEFENSE IS ESTABLISHED ON THE GROUND OF INSANITY.

2. A CRIME CANNOT BE COMMITTED BY THAT DEFENDANT.

When lined up in this manner, the changes in terminology stand out clearly.

M'NAGHTEN

HALL

- 1. is clearly proved
- 2. accused party
- 3. laboring under such a defect of reason
- 4. to know
- 5. nature
- 6. quality
- 7. to know
- 8. what was wrong
- 9. act
- 10. - - - - -
- 11. defect of reason
- 12. disease of the mind
- 13. defense is established

- 1. trier of facts decides
- 2. defendant
- 3. lacked the capacity
- 4. to understand
- 5. physical nature
- 6. consequences
- 7. to realize
- 8. morally wrong
- 9. harm
- 10. forbidden by criminal law
- 11. lack of capacity
- 12. mental disease
- 13. a crime cannot be committed

Such a comparison in systematically-pulverized form shows clearly just to what extent the old rule has been modified in the construction of the suggested new proposal; and, perhaps, enables the reader to evaluate better the claimed advantages of the new rule.

### 13.0 *Simplification*

Another way in which systematic pulverization will be useful is simplifying statements and making them more comprehensible. For example, statutes frequently contain two widely separated implications that taken together express a complication. In such cases systematic pulverization can alert the draftsman to instances where the more simple and comprehensible single complication can be used instead of a pair of implications. Sections 74 and 117 of the Internal Revenue Code of 1954 illustrate this:

*“Section 74 Prizes and Awards*

- (a) *General Rule*—Except as provided in subsection (b) and in section 117 (relating to scholarships and fellowship grants), gross income includes amounts received as prizes and awards.
- (b) *Exception*—Gross income does not include amounts received as prizes and awards made primarily in recognition of religious, charitable, scientific, educational, artistic, literary, or civic achievement, but only if—
  - (1) the recipient was selected without any action on his part to enter the contest or proceeding; and
  - (2) the recipient is not required to render substantial future services as a condition to receiving the prize or award.”

One convenient pulverization of section 74 into its constituent elements would be the following:

- A = it is otherwise provided in 13.4
- B = it is otherwise provided in section 117  
(relating to scholarship and fellowship grants)
- C = amounts are received as prizes &OR awards
- D = such prizes &OR awards are made primarily in recognition of
  - 1) religious
  - 2) &OR charitable
  - 3) &OR scientific
  - 4) &OR educational
  - 5) &OR artistic
  - 6) &OR literary
  - 7) &OR civic
 achievement
- E = there was action on the part of the selected recipient to enter the contest &OR proceeding
- F = the recipient is required to render substantial future services as a condition to receiving the prize &OR award
- G = SUCH AMOUNTS ARE INCLUDED IN GROSS INCOME.



In schematic form section 74 would be represented by the following pair of propositions:

- 13.1      1.      NOT A  
           &2.     NOT B  
           &3.     C  
           -----  
           4.      G
- 13.2      1.      C  
           &2.     D  
           -----  
           3.      1.      NOT E  
                   &2.     NOT F  
                   -----  
                   3.      NOT G

In systematically-pulverized form section 74 would be:

- 13.3      GENERAL RULE  
           1. It is NOT otherwise provided in 13.4  
           &2. it is NOT otherwise provided in section 117  
               (relating to scholarship and fellowship grants)  
           &3. amounts are received as prizes &OR awards  
           -----  
           4. SUCH AMOUNTS ARE INCLUDED IN GROSS  
               INCOME.
- 13.4      EXCEPTION  
           1. Amounts are received as prizes &OR awards  
           &2. such prizes &OR awards are made primarily in recog-  
               nition of  
               1)        religious  
               2)        &OR charitable  
               3)        &OR scientific  
               4)        &OR educational  
               5)        &OR artistic  
               6)        &OR literary  
               7)        &OR civic  
               achievement  
           -----  
           3.      1. there was NOT any action on the part  
                   of the selected recipient to enter the  
                   contest &OR proceeding  
                   &2. the recipient is NOT required to  
                   render substantial future services  
                   as a condition to receiving the prize  
                   &OR award  
                   -----  
                   3. SUCH AMOUNTS ARE NOT INCLUDED  
                   IN GROSS INCOME.

If the constituent elements of these two propositions are examined more closely it is discovered that A is equivalent to the combination of D & NOT E & NOT F. This, in turn, means that 13.1 and 13.2 taken together establish a complication between:

1. D
- &2. NOT G

whenever the other conditions are satisfied. Further examination reveals that

1. B
- &2. section 117

when combined with the other elements in 13.1 and 13.2, establish a complication between

1. B
- &2. NOT G

whenever the other conditions are satisfied. This, in turn, means that the pair of propositions, 13.1 and 13.2, and a portion of section 117 can be represented by the single proposition:

- |      |    |    |       |     |       |
|------|----|----|-------|-----|-------|
| 13.5 | 1. | C  |       |     |       |
|      | 2. | 1. | 1)    | B   |       |
|      |    |    | 2)    | &OR |       |
|      |    |    |       | 1.  | D     |
|      |    |    |       | &2. | NOT E |
|      |    |    |       | &3. | NOT F |
|      |    | 2. | NOT G |     |       |

In systematically-pulverized form:

- 13.6      1. Amounts are received as prizes &OR awards
- 
2. 1. 1)      that amount qualifies for  
                 exclusion in section 117 (relating  
                 to scholarship and fellowship grants)
- 2) &OR 1. such prizes &OR awards are  
                 made primarily in recognition  
                 of
- 1)      religious  
                         2) &OR charitable  
                         3) &OR scientific  
                         4) &OR educational  
                         5) &OR artistic  
                         6) &OR literary  
                         7) &OR civic
- achievement
- &2. there was NOT any action on  
                 the part of the selected  
                 recipient to enter the contest  
                 &OR proceeding
- &3. the recipient is NOT required  
                 to render substantial future  
                 services as a condition to  
                 receiving the prize &OR  
                 award

---

2. SUCH AMOUNTS ARE NOT INCLUDED  
IN GROSS INCOME.

Because the argument indicating the equivalence of proposition 13.6 to the pair of propositions, 13.3 and 13.4, and a portion of section 117 is not spelled out completely,<sup>39</sup> Table I has been constructed to convince those who may be skeptical about this asserted equivalence. There are 32 possible combinations of the antecedent conditions C, B, D, E and F; and Table I shows that for every one of the 32 possible combinations, both 13.6, as expressed in systematically-pulverized form, and section 74 and a portion of section 117, as expressed in the Internal Revenue Code, lead to the same conclusion about G (*i.e.*, about whether the amount shall be included in gross income).

---

39. The only reason for not doing so here is that it would be an extremely complex analysis in words. In symbolic notation the proof of this equivalence runs to approximately 100 steps.

Table 1

					N = NOT		
ANTECEDENT FACTS					CONCLUSION FROM		
					SECTION 74 AND A PORTION OF SECTION 117		PROPOSITION 13.6
1.	NC,	NB,	ND,	NE, NF	----		----
2.	NC,	NB,	ND,	NE, F	----		----
3.	NC,	NB,	ND,	E, NF	----		----
4.	NC,	NB,	ND,	E, F	----		----
5.	NC,	NB,	D,	NE, NF	----		----
6.	NC,	NB,	D,	NE, F	----		----
7.	NC,	NB,	D,	E, NF	----		----
8.	NC,	NB,	D,	E, F	----		----
9.	NC,	B,	ND,	NE, NF	----		----
10.	NC,	B,	ND,	NE, F	----		----
11.	NC,	B,	ND,	E, NF	----		----
12.	NC,	B,	ND,	E, F	----		----
13.	NC,	B,	D,	NE, NF	----		----
14.	NC,	B,	D,	NE, F	----		----
15.	NC,	B,	D,	E, NF	----		----
16.	NC,	B,	D,	E, F	----		----
17.	C,	NB,	ND,	NE, NF	NA	G	G
18.	C,	NB,	ND,	NE, F	NA	G	G
19.	C,	NB,	ND,	E, NF	NA	G	G
20.	C,	NB,	ND,	E, F	NA	G	G
21.	C,	NB,	D,	NE, NF	A	NG	NG
22.	C,	NB,	D,	NE, F	NA	G	G
23.	C,	NB,	D,	E, NF	NA	G	G
24.	C,	NB,	D,	E, F	NA	G	G
25.	C,	B,	ND,	NE, NF	NA	NG	NG
26.	C,	B,	ND,	NE, F	NA	NG	NG
27.	C,	B,	ND,	E, NF	NA	NG	NG
28.	C,	B,	ND,	E, F	NA	NG	NG
29.	C,	B,	D,	NE, NF	A	NG	NG
30.	C,	B,	D,	NE, F	NA	NG	NG
31.	C,	B,	D,	E, NF	NA	NG	NG
32.	C,	B,	D,	E, F	NA	NG	NG

This should persuade even the most dubious that 13.6 is equivalent to section 74 and the relevant portion of section 117 as they are now written in the Internal Revenue Code. In both, every imaginable combination of relevant

facts leads to the same conclusion about whether the amount is to be included in gross income. Yet the systematically-pulverized form shown in 13.6 is a distinct improvement in terms of simplicity and comprehension. And it can be fairly assumed that many sections of the Internal Revenue Code—as well as other statutes—are appropriate candidates for such simplification.

#### CONCLUSION

Although he regarded perfect drafting as unattainable, Justice Cardozo recognized that:

“The task of judicial construction would be easier if statutes were invariably drafted with unity of plan and precision of expression. Indeed, adherence to the same standards would be useful also in opinions.”<sup>40</sup>

It might be that one of the consequences of such improvement in drafting would be to enhance the role of the legislature in the determination of public policy. Lest this is too remindful of past attacks on “judicial legislation” and disappointed hopes for controlling the judiciary by codification, it is freely acknowledged here that some form of judicial legislation is not only inevitable, but also desirable. The important point that should be emphasized is that judicial legislation arises from at least two different sources, only one of which can be justifiably defended.

Judge Frank indicated this distinction in answering Bentham and his disciples on their criticism of the power exercised by judges in construing statutes.<sup>41</sup> To Frank the failure in Europe of repeated attempts to destroy judicial legislation through codification was significant. He regarded as a “fatuous dream” the notions that all policy can come solely from the legislature—that legal certainty can be achieved by using codification to eliminate judicial legislation. According to Frank, when courts interpret statutes, they cannot avoid engaging in supplemental law making, for two reasons:

“[T]he necessary generality in the wording of many statutes, and ineptness in the drafting of others, frequently compels the courts, as best they can, to fill in the gaps, an activity which, no matter how one may label it, is in part legislative.”<sup>42</sup>

Few would dispute this contention. However, Frank’s analysis becomes even more interesting if carried further by pointing out one clear distinction between:

1. the necessary generality in the wording of many statutes
- & 2. ineptitude in drafting.

40. *Finsilver, Still & Moss, Inc. v. Goldberg, Maas & Co.*, 253 N.Y. 382, 392, 171 N.E. 479, 482 (1930).

41. *Guisseppi v. Walling*, 144 F.2d 608 (2d Cir. 1944).

42. *Id.* at 621.

Because of the first, the filling of gaps in legislation by courts cannot and should not be entirely eliminated. However, this is *not* the case with judicial legislation made necessary by ineptitude in drafting. The contention here is that the necessity for judicial legislation should be minimized insofar as that necessity arises from drafting ineptitude. This Article represents an effort to devise techniques to curtail drafting ineptitude and the ambiguities thereby created.

The technique of systematic pulverization is based on the proposition that communication can be clarified by identifying and spotlighting the logical connectives embodied in a message. This brief survey of six of the elementary logical connectives suggests that the extent to which such clarification can be achieved when the full apparatus of symbolic logic is used, may be quite impressive, indeed. Even this initial application of symbolic logic provides significant help in avoiding some of the pervasive problems of legal drafting and interpretation.

Some of the potential virtues inherent in a more general application of symbolic logic to legal thinking are likely to be found in systematic pulverization. In a recent article Professor Ilmar Tamello has opened the door on discussion of such a general application of symbolic logic as a tool for legal analysis.<sup>43</sup> Although he asserts more than he demonstrates and commits some important technical errors,<sup>44</sup> many of his observations deserve close scrutiny and further investigation. His intuitions about the usefulness of symbolic logic as a tool for legal analysis, which may well turn out to be sound, seem pertinent enough to include here in detail. The following is a slightly paraphrased summary of his observations:

- (1) Logic in general can be used as a universal form of reference, an all-embracing theory of scientific research to coordinate all of the different disciplines.
- (2) Symbolic logic is more exact and more comprehensive than traditional logic.
- (3) Even though traditional logic is more easily communicated in our present state of learning because of its greater familiarity, nevertheless symbolic logic is capable of more effective rational penetration.
- (4) The increasing complexity of legal analysis, just as in any other discipline, produces a greater need for the simplification and precision of symbolic logic; and the tendency away from traditional logic is already visible in philosophy, natural sciences and theoretical economy.
- (5) Symbolic logic is *not* a new conception of law; it is *not* a source of experience, but only an intellectual tool to master human experience.
- (6) Symbolic logic employed in legal thinking will *not* deny the role of intu-

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43. Tamello, *Sketch for a Symbolic Juristic Logic*, 8 J. LEGAL ED. 277, 302-06 (1956). See also Anderson & Moore, *The Formal Analysis of Normative Concepts*, 22 AM. SOC. REV. 9 (1957).

44. Clark, *On Mr. Tamello's Conception of Juristic Logic*, 8 J. LEGAL ED. 491 (1956).

ition; there is room in legal thinking for both analytical and intuitive approaches.

- (7) Exponents of symbolic juristic logic are *not* seeking to promote legal dogmatism by means of a "super logic"; on the contrary, they suggest that it provides a logical means of penetration into the sociological substratum of law, and excels traditional logic in doing so.

Recognizing that misunderstanding about the suggested use of symbolic logic as a tool for legal analysis will arise quite naturally, Tamello suggests:

"The still hazy outlines of a new province of knowledge emerging in the field of our intellectual vision provide a ground onto which we naturally tend to project our scholarly sympathies and antipathies, expectations and apprehensions. Recognition of the fact that the projection mechanism of our mind operates also in our scholarly pursuits should permit us to be patient with new developments of learning until they have had a chance to expose their true virtues and limitations."<sup>45</sup>

A practical illustration of how symbolic logic can be employed in a legal context was set forth more than six years ago in an article by John Pfeiffer, in which he described the use of symbolic logic to rewrite a provision in a contract of the Prudential Life Insurance Company. He stated:

"Symbolic logic has since been used in many other insurance problems. Mathematicians at Equitable, Metropolitan, Aetna and other companies have applied it to the analysis of war clauses and employee eligibility under group contracts. And other corporations have found symbolic logic very helpful in analyzing their contracts. Contracts between large corporations may run into many pages of fine print packed with stipulations, contingencies and a maze of ifs, ands and buts. Are the clauses worded as simply as they might be? Are there loop-holes or inconsistencies? A symbolic analysis can readily answer such questions, and lawyers have begun to call on mathematicians to go over their contracts."<sup>46</sup>

To a profession whose most important skill is the manipulation of verbal symbols, further inquiry would seem to be clearly warranted—inquiry into the significance and relevance of this powerful analytic tool for purposes of legal analysis. Its significance for drafting and interpretation alone ought to be enough to justify serious consideration of including some formal instruction in symbolic logic as part of law school training.

45. Tamello, *supra* note 43, at 304.

46. Pfeiffer, *Symbolic Logic*, *Scientific American*, Dec. 1950, pp. 22, 23, 24.

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