

# Book Review

## Naked Statistical Evidence

*Quantitative Methods in Law: Studies in the Application of Mathematical Probability and Statistics to Legal Problems.* By Michael Finkelstein. New York: Free Press, 1978. Pp. xi, 318. \$15.00.

### Reviewed by David Kaye†

Statistical methodology has become an increasingly important tool in a surprising variety of disciplines. Business majors are exposed to econometrics. Medical students are taught biostatistics. Even education majors are confronted by psychometrics. Lawyers and law students, however, have remained remarkably unaffected by and ignorant of formal statistical thinking. "Jurimetrics" is a word coined by a few political scientists who, by brute empiricism, generate mathematical models of judicial decisions.<sup>1</sup> No more than a handful of law schools have offerings in probability or statistics, and it is an unusual lawyer who knows that regression analysis is not available from his psychoanalyst.

In these circumstances, Michael Finkelstein is something of an anomaly. A New York lawyer and an adjunct professor at Columbia University, Finkelstein has tirelessly advocated the use of statistical techniques in legal analysis and decisionmaking. His early work on inferring the likelihood of jury discrimination on the basis of a binomial probability distribution model<sup>2</sup> has become an accepted tool in resolving claims of discrimination in grand jury selection.<sup>3</sup> His plea

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1. See, e.g., F. KORT, *A SPECIAL AND A GENERAL MULTIVARIATE THEORY OF JUDICIAL DECISIONS* (1976); G. SCHUBERT, *HUMAN JURISPRUDENCE* (1974).

2. Finkelstein, *The Application of Statistical Decision Theory to the Jury Discrimination Cases*, 80 HARV. L. REV. 338 (1966).

3. See, e.g., *Castaneda v. Partida*, 430 U.S. 482, 485-99 (1977) (statistics on jury composition held sufficient to establish prima facie case of intentional discrimination).

for the incorporation of Bayesian analysis in the evaluation of quantitative identification evidence<sup>4</sup> has found less favor among courts and commentators,<sup>5</sup> but it has stimulated the most lively and penetrating discussions of statistical proof to date.<sup>6</sup>

In *Quantitative Methods in Law*,<sup>7</sup> Finkelstein collects these and other essays and presents them in an engaging and updated format. The additional topics discussed include the application of the "one-man, one-vote" reapportionment doctrine to multimember districts, the quantity of voting irregularities necessary to mandate a rerun of a challenged primary election, the measurement of market concentration as a guide for merger policy, the extent of minimum reserves insurance companies should be required to maintain, and the use of econometric models in administrative proceedings. No prior background in statistics is assumed, for the volume is directed to lawyers, not mathematicians. To the extent that its aim is to persuade this audience that statistical tools have their place in litigation, administrative proceedings, and legal analysis, the book is, I think, eminently successful.

This is not to say, however, that every quantitative technique of value to the law is studied or even that the limitations of the techniques considered are critically delineated.<sup>8</sup> The book is not intended and would not serve as a primer on statistics for lawyers. It is best read as a sampler, intended to whet the palate and to illustrate some basic ideas. The approach is often both original and idiosyncratic, and the conclusions controversial even among partisans of courtroom mathematics. This review will examine one of Finkelstein's more dubious arguments. I should hasten to add, however, that his analysis is novel, stimulating, and pertains to a fundamental problem in the law of

4. Finkelstein & Fairley, *A Bayesian Approach to Identification Evidence*, 83 HARV. L. REV. 489 (1970).

5. See, e.g., Brilmayer & Kornhauser, *Review: Quantitative Methods and Legal Decisions*, 46 U. CHI. L. REV. 116 (1978); Lempert, *Modeling Relevance*, 75 MICH. L. REV. 1021 (1977). But see Wagner, Book Review, 1979 DUKE L.J. 1071; Ellman & Kaye, *Probabilities and Proof: Can Blood Group Testing Prove Paternity?* (forthcoming 55 N.Y.U. L. REV. (1980)).

6. See Tribe, *Trial by Mathematics: Precision and Ritual in the Legal Process*, 84 HARV. L. REV. 1329 (1971) [hereinafter cited as *Trial by Mathematics*]; Finkelstein & Fairley, *A Comment on "Trial by Mathematics"*, 84 HARV. L. REV. 1801 (1971); Tribe, *A Further Critique of Mathematical Proof*, 84 HARV. L. REV. 1810 (1971); Fairley, *Probabilistic Analysis of Identification Evidence*, 2 J. LEGAL STUD. 493 (1973); Brilmayer & Kornhauser, *supra* note 5; Gerjuoy, *The Relevance of Probability Theory to Problems of Relevance*, 18 JURIMETRICS J. 1 (1977); Kaye, *The Laws of Probability and the Law of the Land* (forthcoming 46 U. CHI. L. REV. (1979)).

7. M. FINKELSTEIN, *QUANTITATIVE METHODS IN LAW: STUDIES IN THE APPLICATION OF MATHEMATICAL PROBABILITY AND STATISTICS TO LEGAL PROBLEMS* (1978) [hereinafter cited by page number only].

8. See Brilmayer & Kornhauser, *supra* note 5.

evidence. Indeed, the point he raises has even prompted one philosopher and a few academic lawyers to conclude that probability theory must be radically reformulated if it is to account for the rules governing legal fact-finding.<sup>9</sup>

The problem is the attempt to articulate, in starkly quantitative terms, the burden of proof in civil cases. Hornbook law has it that in most civil cases, the plaintiff must establish all the essential elements of his case by a preponderance of the evidence.<sup>10</sup> A majority of courts and almost all commentators have concluded that this standard is satisfied by evidence that indicates to the trier of fact that the event that must be established is more likely to have occurred than not.<sup>11</sup> Nevertheless, this "more-probable-than-not" interpretation has precipitated acute discomfort when applied to what might be called naked statistical evidence. In *Sargent v. Massachusetts Accident Company*,<sup>12</sup> for instance, the Supreme Judicial Court of Massachusetts, while adopting a "greater-chance-than-not" test, nevertheless insisted that it is

not enough that mathematically the chances somewhat favor a proposition to be proved; for example, the fact that colored automobiles made in the current year outnumber black ones would not warrant a finding that an undescribed automobile of the current year is colored and not black, nor would the fact that only a minority of men die of cancer warrant a finding that a particular man did not die of cancer.<sup>13</sup>

This well-known dictum<sup>14</sup> is the starting point for Finkelstein's treatment of the conventionally accepted more-probable-than-not standard. He first observes, quite correctly, that this standard has the virtue of minimizing the total number of incorrect verdicts. To illus-

9. Other arguments are also adduced in support of this conclusion. See L. COHEN, *THE PROBABLE AND THE PROVABLE* (1977); Brilmayer & Kornhauser, *supra* note 5. This thesis is criticized in, e.g., Kaye, *supra* note 6; Schum, Book Review, 77 MICH. L. REV. 446 (1979).

10. See, e.g., C. McCORMICK, *EVIDENCE* 793-94 (2d ed. 1972). In certain types of civil cases, a heavier burden may be imposed on the plaintiff. See, e.g., *Addington v. Texas*, 441 U.S. 418, 431-33 (1979) (constitutional requirement of clear and convincing evidence in civil commitment proceeding). In addition, certain evidentiary rules may shift the burden from plaintiff to defendant in some circumstances. See, e.g., *Weiss v. Axler*, 137 Colo. 544, 328 P.2d 88 (1958) (minority position on procedural effect of *res ipsa loquitur*).

11. E.g., R. EGGLESTON, *EVIDENCE, PROOF AND PROBABILITY* 109 (1978); C. McCORMICK, *supra* note 10, at 794.

12. 307 Mass. 246, 29 N.E.2d 825 (1940).

13. *Id.* at 250, 29 N.E.2d at 827.

14. See, e.g., Ball, *The Moment of Truth: Probability Theory and Standards of Proof*, 14 VAND. L. REV. 807, 818 (1961); Winter, *The Jury and the Rule of Nonpersuasion*, 5 LAW & SOC'Y REV. 335, 338 (1971).

trate the logic of his argument, suppose that in a class of cases tried to the court, plaintiffs must prove some disputed fact *X* in order to prevail. Imagine that a numerically minded judge, after hearing all of the evidence pertaining to *X* in each of these cases arising in a given year, writes his best estimate of the probability of *X* in a small black notebook. In a few rather shoddy cases, the  $p(X)$  he records is close to zero. In some unusually compelling cases,  $p(X)$  approaches one. In most instances,  $p(X)$  falls somewhere between these two extremes.

Now suppose that the judge asks us to formulate a rule to tell him when the probability is high enough to justify resolving *X* in plaintiff's favor. He reveals that he is interested only in making correct decisions and avoiding incorrect ones—correct in the sense that, in the long run, the decisions will correspond to the true state of affairs as often as possible. Finally, he gives us the secret notebook in which he has recorded the year's worth of probability estimates.

We inspect the notebook and rearrange the probability estimates to form a table showing the number of times (*n*) that the probability takes on various values.<sup>15</sup> In this way we uncover the following pattern:

p	.1	.2	.3	.4	.5	.6	.7	.8	.9
n	0	0	2	6	10	20	20	60	30

That is, of all the cases heard, the probability in favor of plaintiff was never .1 or .2. It was .3 in two instances, .4 in another six cases, and so on. If the judge's estimates are good, so that we can take them as accurate statements of the probability of *X*,<sup>16</sup> we can speak of the expected number of correct decisions under various decision rules.<sup>17</sup>

For example, in the twenty cases in which *p* is .6 we should expect  $20 \times .6 = 12$  of them to be cases in which the disputed fact *X* was as plaintiff claimed. On this basis, if the judge decided all twenty cases in favor of plaintiff, his expected number of correct decisions would be twelve, and the expected number of mistakes would be eight. Plainly, if mistakes are to be minimized, these cases should be decided for

15. In this illustration *p* is taken to be discrete for ease of exposition. In the footnotes that generalize this example, *p* is allowed to be continuous on the interval [0,1], and *n*(*p*) is akin to a probability density function.

16. Some commentators have questioned whether such an assumption can be made. Brillmayer & Kornhauser, *supra* note 5, at 141.

17. According to a corollary of the statistician's "law of large numbers," the actual number of correct decisions approaches this expected number in the limit as the number of cases approaches infinity. See, e.g., H. BRUNK, AN INTRODUCTION TO MATHEMATICAL STATISTICS 153 (Corollary C) (3d ed. 1975). In common parlance, this is known as the "law of averages."

plaintiffs. Pursuing this logic to the bitter end reveals that the judge makes the least mistakes if he adopts the following decision rule: resolve  $X$  in plaintiff's favor when  $p$  is greater than one-half; otherwise, find in defendant's favor.<sup>18</sup> This result, furthermore, is not an artifact of the numerical example given here. It is quite general and holds for distributions of any shape.<sup>19</sup>

The more basic question is why minimization of the total number of errors is the *summum bonum* of civil litigation. Finkelstein poses this issue and purportedly resolves it by insisting that there is a competing goal of "equalization," or allocating errors to plaintiffs and defendants in roughly equal proportions. This goal, Finkelstein suggests, often leads to a more stringent standard than the "more-probable-than-not" test. In this way he defends the dictum in *Sargent*, concluding that

18. Another decision rule that works just as well prescribes a decision for plaintiff whenever  $p$  equals or exceeds .5. The rule in the text incorporates the accepted legal convention of deciding issues in which  $p(X) = .5$  in favor of defendant.

19. Let  $n(p)$  be the frequency distribution of the probabilities  $p$  over the  $N$  cases decided in some time period. Then

$$N = \int_0^1 n(p) dp.$$

Under a decision rule that mandates decisions for plaintiff whenever  $p > p_0$ , the expected number of erroneous decisions for plaintiffs will be

$$N_{\circ\pi} = \int_{p_0}^1 (1-p)n(p) dp.$$

The expected number of erroneous decisions for defendants is given by

$$N_{\circ\Delta} = \int_0^{p_0} pn(p) dp$$

To find the value of  $p_0$  that minimizes the expected total number of errors  $N_{\circ} = N_{\circ\pi} + N_{\circ\Delta}$ , we set  $dN_{\circ}/dp_0 = 0$ , solve for  $p_0$ , and find that the critical value of  $p_0$  is indeed .5.

One can prove more generally that the  $p > .5$  decision rule is error-minimizing by considering stochastic decision rules. Let a general decision rule be specified by a function  $\delta(p)$  on the domain  $[0,1]$  and with range  $[0,1]$ , where  $\delta(p)$  is the probability of finding for plaintiff given the value of  $p$ ; that is, in a case involving evidence such that there is a probability  $p$  that plaintiff is right, it can be understood to say that the jury should decide which way to rule by drawing from an urn in which a fraction  $\delta(p)$  of the lots are for plaintiffs. The error-minimization problem is then to minimize

$$\begin{aligned} N_{\circ} &= N_{\circ\pi} + N_{\circ\Delta} = \int_0^1 (1-p)n(p)\delta(p) dp + \int_0^1 pn(p)[1-\delta(p)] dp \\ &= \int_0^1 pn(p) dp + \int_0^1 (1-2p)n(p)\delta(p) dp \end{aligned}$$

The first integral does not depend on  $\delta(p)$ ; the second is minimized by that function which minimizes  $[(1-2p)n(p)\delta(p)]$  independently for each value of  $p$ . This will mean choosing  $\delta(p)$  as large as possible when  $(1-2p)$  is negative, and as small as possible when  $(1-2p)$  is positive. Thus, resolving cases in which  $p = .5$  in favor of defendants as in note 18 *supra*, the optimal decision rule is

$$\delta(p) = \begin{cases} 1 & \text{if } p > .5 \\ 0 & \text{if } p \leq .5 \end{cases}$$

This is, of course, the  $p > .5$  decision rule.

“the rejection of statistical evidence which establishes a proposition by a preponderance of probabilities is not necessarily inconsistent with the preponderance of evidence standard, since that standard may involve a higher level of proof if the decision maker seeks to equalize errors.”<sup>20</sup>

How then does this criterion of “equalization” operate? By “equalization” Finkelstein appears to mean making the same expected number of erroneous decisions for plaintiffs as for defendants.<sup>21</sup> This is indeed a very different criterion than minimization of errors. There are many distributions of  $p$  in which the more-probable-than-not ( $p > .5$ ) decision rule will lead, as Finkelstein remarks, to a larger incidence of erroneous decisions in favor of plaintiffs as opposed to defendants (or vice versa).<sup>22</sup>

Consider, for example, the probabilities estimated by our imaginary judge. Deciding by the more-probable-than-not rule yields 130 decisions for plaintiffs and only eighteen for defendants. The expected number of errors favoring defendants will be

$$(.1 \times 0) + (.2 \times 0) + (.3 \times 2) + (.4 \times 6) + (.5 \times 10) = 8,$$

and the number favoring plaintiffs will be

$$(1-.6) \times 20 + (1-.7) \times 20 + (1-.8) \times 60 + (1-.9) \times 30 = 29.$$

The  $p > .5$  rule is thus very one-sided. Nearly four-fifths of the errors accrue to the benefit of plaintiffs. Following Finkelstein’s suggestion, we would enhance “equality” by moving up from  $p > .5$  to, say,  $p > .65$ . That is, the judge would decide for defendant whenever his probability estimate is .65 or less and for plaintiff only when the case is especially strong, so that the estimate exceeds .65. Under this rule, there will be twenty expected errors favoring defendants and twenty-

20. P. 69.

21. Mistakes, he says, “must come in pairs and be equal in number.” P. 68. In the terminology of note 19 *supra*, this is to say that

$$N_{\sigma\pi} = N_{\sigma\Delta}, \text{ i.e., } \int_{p_0}^1 (1-p)n(p)dp = \int_0^{p_0} pn(p)dp.$$

One might equally well have supposed that the criterion for “equality” should be

$$N_{\sigma\pi}/N_{\pi} = N_{\sigma\Delta}/N_{\Delta}, \text{ where } N_{\pi} = \int_{p_0}^1 n(p)dp \text{ and } N_{\Delta} = \int_0^{p_0} n(p)dp.$$

22. When  $n(p)$  is symmetrical about  $p = .5$ , the  $p > .5$  rule not only maximizes the total number of expected correct decisions, but also assigns the correct decisions to plaintiffs and defendants in equal numbers (and, for that matter, in proportion to the number of decisions for plaintiffs and defendants, respectively). Since the function  $1-p$  (for  $.5 < p < 1$ ) is the mirror image of the function  $p$  (for  $0 < p < .5$ ) about the line  $p = .5$ , it follows that  $N_{\sigma\pi} = N_{\sigma\Delta}$  if  $n(p)$  is also symmetric about .5.

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one favoring plaintiffs. In sum, “equalization” here calls for a more demanding standard of proof and entails more errors, but when it errs, it errs in a more “equal” way. Nor is this picture unique to this example. As long as the center of the distribution tends toward values favoring plaintiffs, “equalization” can be achieved only by deciding  $X$  for plaintiffs when  $p(X)$  is rather larger than .5, thereby increasing the gross number of errors.<sup>23</sup>

Nevertheless, in invoking this criterion of “equalization” to explain the reluctance of courts to permit proof by naked statistical evidence, Finkelstein is enticed into a diverting but ultimately unsatisfying frolic. To begin with, the same logic can be used not to justify, but to criticize the *Sargent* dictum. If the distribution of the probabilities peaks at or leans toward some central value below one-half, then “equalization” requires that plaintiffs prevail on the basis of statistical (or other) evidence that flunks even the minimal  $p > .5$  standard. In these situations Finkelstein’s argument suggests that courts should welcome still weaker statistical evidence than that which was frowned upon in *Sargent*.<sup>24</sup>

More fundamentally, why should we care about “equalization” in the first place? Mistakes do not cancel one another out: it is no solace to the defendant who should have prevailed but did not that somewhere there is or will be a similarly affected plaintiff. Unless plaintiffs and defendants are different sorts of people such that defendants deserve to be favored, I cannot imagine why we should seek this sort of “equality.” There is nothing unfair in telling any defendant that he is liable to plaintiff because we are more convinced that the truth lies on the side of plaintiff than defendant. Indeed, to deny recovery to plaintiffs in these circumstances seems the greater injustice. What are we to say to these plaintiffs? “Ordinarily you would have won, for your

23. For  $N_{\circ}\pi$  to equal  $N_{\circ}\Delta$  under a decision rule that specifies decisions for plaintiffs whenever  $p > p_{\circ}$ , the following equality must hold:

$$\int_{p_{\circ}}^1 (1-p)n(p)dp = \int_0^{p_{\circ}} pn(p)dp$$

Expanding the first integral and rearranging terms, we find

$$\int_{p_{\circ}}^1 n(p)dp = N \int_0^1 p \frac{n(p)}{N} dp = N\bar{p}$$

Thus, for a given distribution  $n(p)$ , the value of  $p_{\circ}$  that achieves “equalization” is not generally .5. Rather,  $p_{\circ}$  is such that  $n(p)$  integrated from  $p_{\circ}$  to 1 equals the total number of cases multiplied by the mean value of  $p$  for the given distribution.

24. Finkelstein implies that there is reason to believe that the distributions  $n(p)$  almost always peak or are skewed to the right of  $p = .5$ . P. 78. Since no such reasons are stated, it remains less than obvious that “equalization” affects the choice of  $p_{\circ}$  in the way Finkelstein desires.

case was quite strong; however, because of the large number of even stronger plaintiffs' cases this year, we had to award a few decisions to otherwise undeserving defendants!"

Moreover, Finkelstein himself starts from a premise that precludes his reliance on this formalistic "equalization" criterion, even as a minor supplement to the more basic minimization principle. He explicitly agrees with other writers<sup>25</sup> that "the burden of an error is deemed to be the same for both parties."<sup>26</sup> By this, I take it he means that the disutility of an incorrect decision for one party is the same as that of an incorrect decision for the other and, by extension, that the utilities of correct decisions are similarly related.<sup>27</sup> Since a rational decision-maker will act to maximize expected utility, and because the  $p > .5$  decision rule achieves maximization,<sup>28</sup> it follows that the more-probable-than-not standard does incorporate the only meaningful principle of equality between plaintiffs and defendants—and the one principle Finkelstein himself accepts as a basic premise.

It thus appears that Finkelstein's novel attempt to escape the problem of naked statistical evidence posed in the *Sargent* court's dictum fails. Is the *Sargent* court simply wrong in its treatment of statistical

25. Finkelstein quotes from Vaughn Ball's *The Moment of Truth: Probability Theory and Standards of Proof*, *supra* note 14, at 817. P. 66. For other treatments of this point, see, e.g., Cullison, *Probability Analysis of Judicial Fact-Finding: A Preliminary Outline of the Subjective Approach*, 1969 U. Tol. L. Rev. 538; Kaplan, *Decision Theory and the Fact-finding Process*, 20 STAN. L. REV. 1065 (1968); Lempert, *supra* note 5; *Trial by Mathematics*, *supra* note 6. The Supreme Court has also opined that under the preponderance-of-the-evidence standard the litigants "share the risk of error in roughly equal fashion." *Addington v. Texas*, 441 U.S. 418, 423 (1979).

26. P. 67. Without mentioning the work of Cullison, Kaplan, Tribe, *supra* note 25, and Lempert, *supra* note 5, all of whom show that equating the burden of errors for plaintiffs and defendants implies the  $p > .5$  decision rule, Finkelstein makes the unsupported assertion that "[i]f the burden of errors is deemed to be the same for both parties, the aim should be equal rates of errors for both parties over some assumed class of cases." P. 67. In so doing, Finkelstein seems to confuse the "burden" (or, better, disutility) of mistakes for individual litigants with some sort of aggregate "burden."

27. On the meaning of "utility" in this context, see, e.g., W. BAUMOL, *ECONOMIC THEORY AND OPERATIONS ANALYSIS* 420-36 (4th ed. 1977); J. VON NEUMANN & O. MORGENTHAU, *THEORY OF GAMES AND ECONOMIC BEHAVIOR* 15-31 (3d ed. 1955); Kaye, *Playing Games with Justice: Rawls and the Maximin Rule* (forthcoming in 6 SOC. THEORY & PRAC. (1980)).

28. If the utility of a correct decision (whether for plaintiff or for defendant) is  $U_c$  and the utility of an incorrect decision (whether for plaintiff or for defendant) is  $U_e$ , then a rule prescribing decision for plaintiff whenever  $p > p_0$  will result in the following aggregate utility:

$$U = U_c(N - N_{e\pi} - N_{e\Delta}) + U_e(N_{e\pi} + N_{e\Delta}) = U_c N + (U_e - U_c)N_e.$$

Assuming  $U_c > U_e$ , aggregate utility will therefore be maximized by whatever rule minimizes  $N_e$ . As indicated in note 19 *supra*, this rule is the  $p > .5$  standard.

evidence?<sup>29</sup> Must we abandon the usual interpretation of the preponderance-of-the-evidence standard? Or is it the axioms of probability theory that are in need of repair?<sup>30</sup> I wish to argue for the orthodox view that a party shoulders its burden of proof by a preponderance of the evidence by persuading the finder of fact that the probability in question exceeds one-half. Yet, the *Sargent* court is essentially correct in its claim that statistical evidence involving numbers larger than one-half is often insufficient to warrant a finding for a proponent of purely statistical evidence.<sup>31</sup>

My argument is most easily developed with the aid of the subjective interpretation of probability.<sup>32</sup> To a mathematician, probability is a number assigned to a set or subset of events. To qualify as a probability, such numbers must conform to a few simple axioms from which the usual rules associated with flipping fair coins or drawing cards from a well-shuffled deck can be derived. For example, these mathematical probabilities are such that if the probability of an event  $X$  is denoted  $p(X)$ , then the probability of the complementary event not- $X$  is  $1-p(X)$ .

Although this explication of probability theory may provide an internally consistent system for calculating probabilities, it cannot tell us what the probability numbers really stand for. Attempts to answer this question have generated two main schools of thought.<sup>33</sup> Objective interpretations of probability hold that the numbers are premised either on mutually exclusive, equally likely elementary events, or on the relative frequencies with which observed events occur. In contrast, the subjective view attempts to quantify the strength of belief held by a rational person concerning the occurrence of events. Since the factual issues disputed at trial ordinarily do not constitute equally

29. See Winter, *supra* note 14 (criticizing the *Sargent* dictum as a departure from the proper  $p > .5$  standard).

30. See p. 603 *supra*; Brilmayer & Kornhauser, *supra* note 5, at 141-46; Nesson, *Reasonable Doubt and Permissive Inferences: The Value of Complexity*, 92 HARV. L. REV. 1187, 1199 n.27 (1979).

31. In what follows, I draw on the articles cited in note 25 *supra*, and especially on Tribe, *Trial by Mathematics*, *supra* note 6. I part company, however, with Glanville Williams, who accepts the orthodox interpretation by an argument that commits him to a view that, he concedes, is "[n]o doubt . . . illogical." Williams, *The Mathematics of Proof I*, CRIM. L. REV., May 1979, at 305.

32. Finkelstein discusses the nature of probability at pp. 63-65. Generalizing from a deterministic model of coin flipping, Finkelstein suggests that there is no fundamental difference between the objective and subjective interpretations. P. 64. This claim is disputed by Brilmayer & Kornhauser, *supra* note 5, at 140, although their attack is premised on a rather uncharitable reading of Finkelstein's suggestion.

33. See generally, e.g., V. BARNETT, *COMPARATIVE STATISTICAL INFERENCE* 62-88 (1973); T. FINE, *THEORIES OF PROBABILITY: AN EXAMINATION OF FOUNDATIONS* (1973).

likely, mutually exclusive events and do not repeat themselves so as to produce relative frequencies, this subjective or "personalistic" theory is more suitable for modeling litigation.<sup>34</sup> Subjective probabilities can be shown to obey the usual probability axioms as long as the person's risk-related preferences obey a few plausible postulates.<sup>35</sup>

This distinction between objective and subjective interpretations of probability is particularly helpful to the conundrum posed by naked statistical evidence. The opinion in *Sargent* apparently presupposes that the probabilities determined "objectively" from automobile manufacturers' records or from death certificates must be taken to be the subjective probabilities in question. Suppose, to make the *Sargent* dictum a bit more concrete, that a life insurer has excluded coverage for death caused by cancer, that the insured has died, and that forty-five percent of American males die from cancer. The opinion in *Sargent* seems to suggest that the probability that a particular deceased did not die from cancer is  $1 - .45 = .55$ . But this does not necessarily follow, for at least two reasons. First, it may be appropriate as a matter of policy to treat the subjective probability as less than one-half, and therefore insufficient to support a finding that the insured did not die from cancer, simply to "create an incentive for plaintiffs to do more than establish the background statistics."<sup>36</sup> Second, in addition to observing that forty-five percent of American males die from cancer, a fact finder can legitimately consider the fact that plaintiff is relying on this statistical generalization and nothing more. Unless there is some satisfactory explanation for plaintiff's failure to do more than present gross statistics, a rational judge or juror might well arrive at a subjective probability of less than one-half that the insured did not die of cancer.<sup>37</sup> Hence, there is no need to modify the standard

34. *But see* Kaye, *supra* note 6, for an attempt to apply an objective interpretation.

35. *See, e.g.*, L. SAVAGE, FOUNDATIONS OF STATISTICS (2d ed. 1972). For an illustration of how such subjective belief can be quantified, *see, e.g.*, *Trial by Mathematics*, *supra* note 6, at 1347. An engaging and compact sketch of the historical development of some of the subjective theories can be found in H. RAIFFA, DECISION ANALYSIS 273-78 (1968).

36. *Trial by Mathematics*, *supra* note 6, at 1349. Or, one might say, to further the policy of minimizing the total number of errors (and maximizing utility), the threshold probability must be raised above one-half in cases of naked statistical evidence, since this will encourage the production of more particularized and revealing evidence.

37. This last point can be shown to follow directly from the mathematics of probability theory. All that is required is an elementary version of Bayes's formula that indicates how new information affects a previously established probability. Denoting the "prior probability" as  $p(X)$  and the "posterior probability" as  $p(X/E)$ , the formula states:

$$p(X/E) = \frac{p(E/X)p(X)}{p(E/X)p(X) + p(E/\text{not-}X)p(\text{not-}X)}$$

Suppose the fact finder accepts the statistic about the incidence of cancer fatalities at face value. For him, the subjective probability based on this evidence alone (the "prior prob-

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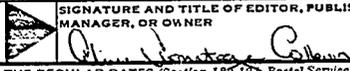
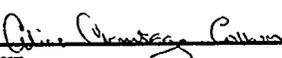
of proof (the  $p > .5$  decision rule), as Finkelstein suggests, to meet the concern raised in *Sargent*.

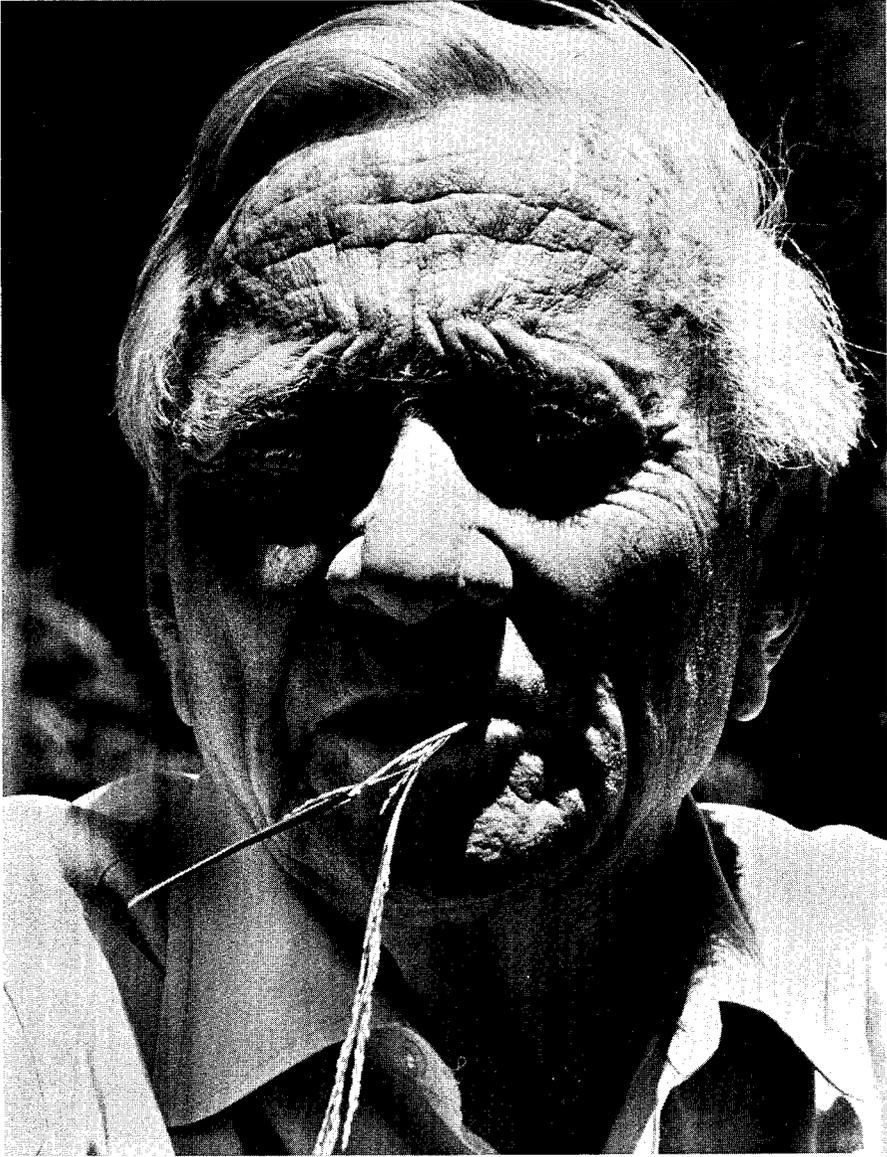
As I noted at the outset, I have focused on what I believe to be one of the weaker points in *Quantitative Methods in Law*. My discussion has shown that the generally accepted quantitative interpretation of the preponderance-of-the-evidence standard ( $p > .5$ ) can be reconciled with the intuitively compelling skepticism of naked statistical evidence voiced in *Sargent v. Massachusetts Accident Company*. In the process, I have demonstrated that under this standard the litigants, as the Supreme Court has recently observed, do indeed “share the risk of error in roughly equal fashion.”<sup>38</sup> But even if Finkelstein’s “equalization” criterion is not required to understand the preponderance-of-the-evidence standard, there is much to ponder and to learn in *Quantitative Methods*. The book deserves a wider audience than it probably will receive.

ability”) that the insured died from something other than cancer is .55. But if he stops to reflect on the fact that this is all there is to the case, he should revise this probability in light of this new item of “evidence,” *E*—the gap in plaintiff’s case. In particular, according to Bayes’s formula, the revised subjective probability  $p(X/E)$  will be one-half or less as long as it is merely 1.22 times more likely that plaintiff would have been able to come forward with more evidence about how the insured died if he had in fact died from cancer than if he had not. See Kaye, *The Paradox of the Gatecrasher and Other Stories*, 1979 ARIZ. ST. L.J. 101, 106 & 108 n.36; Kaye, *Probability Theory Meets Res Ipsa Loquitur*, 77 MICH. L. REV. 1456, 1475-81 (1979).

38. See note 25 *supra*.

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